Threshold ECDSA in Three Rounds

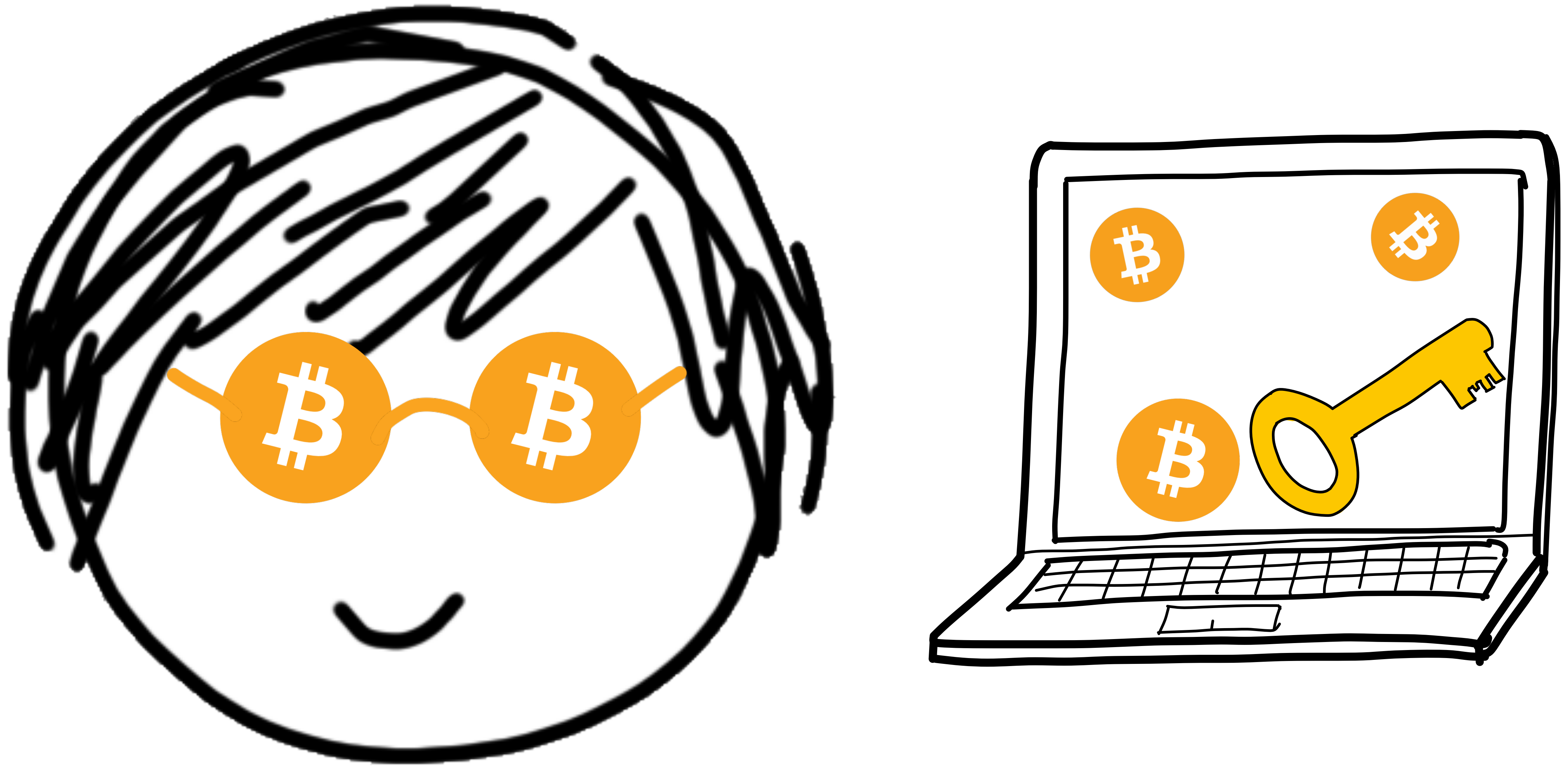
Jack Doerner   Yashvanth Kondi   Eysa Lee   abhi shelat

TECHNION Israel Institute of Technology

AARHUS UNIVERSITY

Northeastern University

Ballad of ₿itcoin Bob



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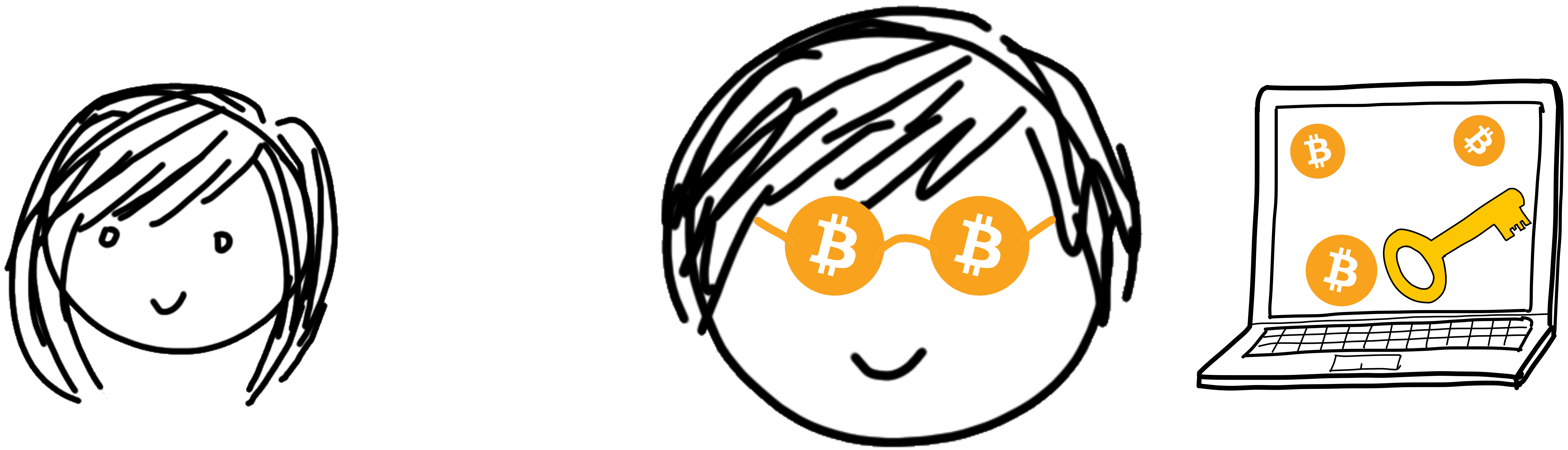

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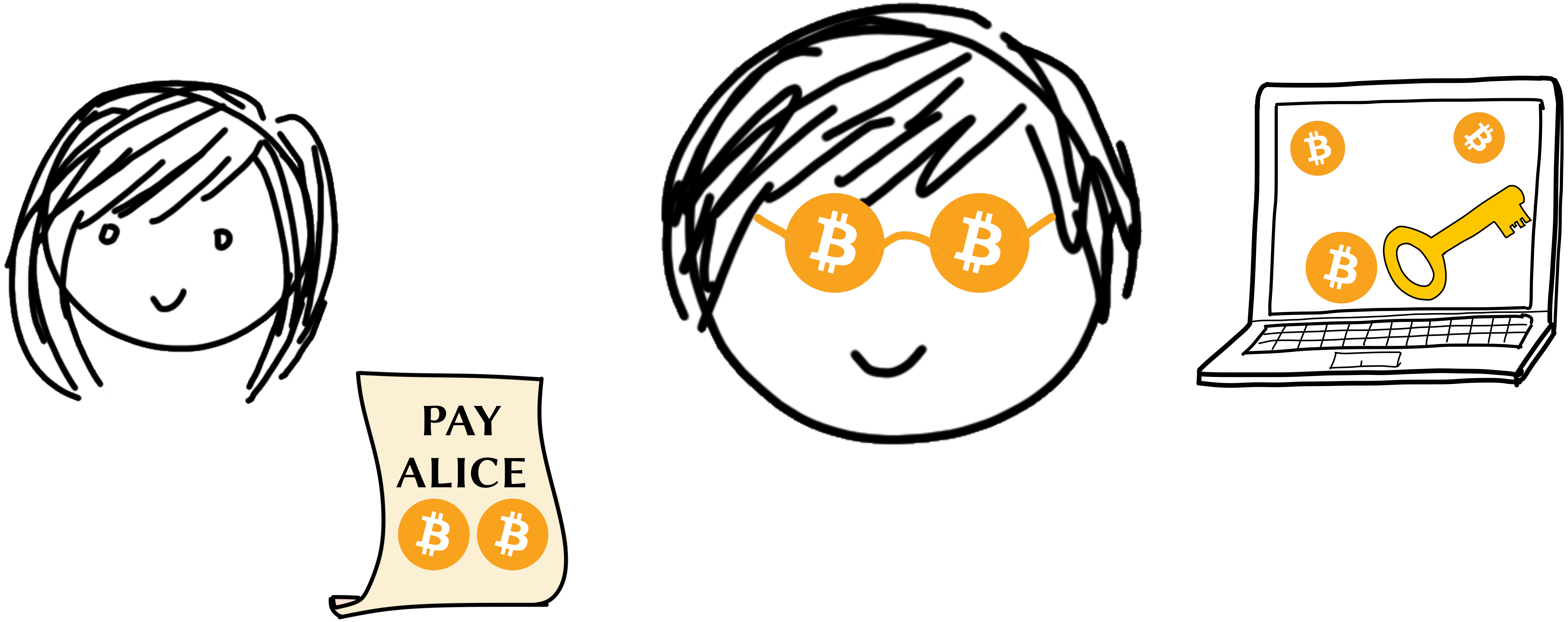

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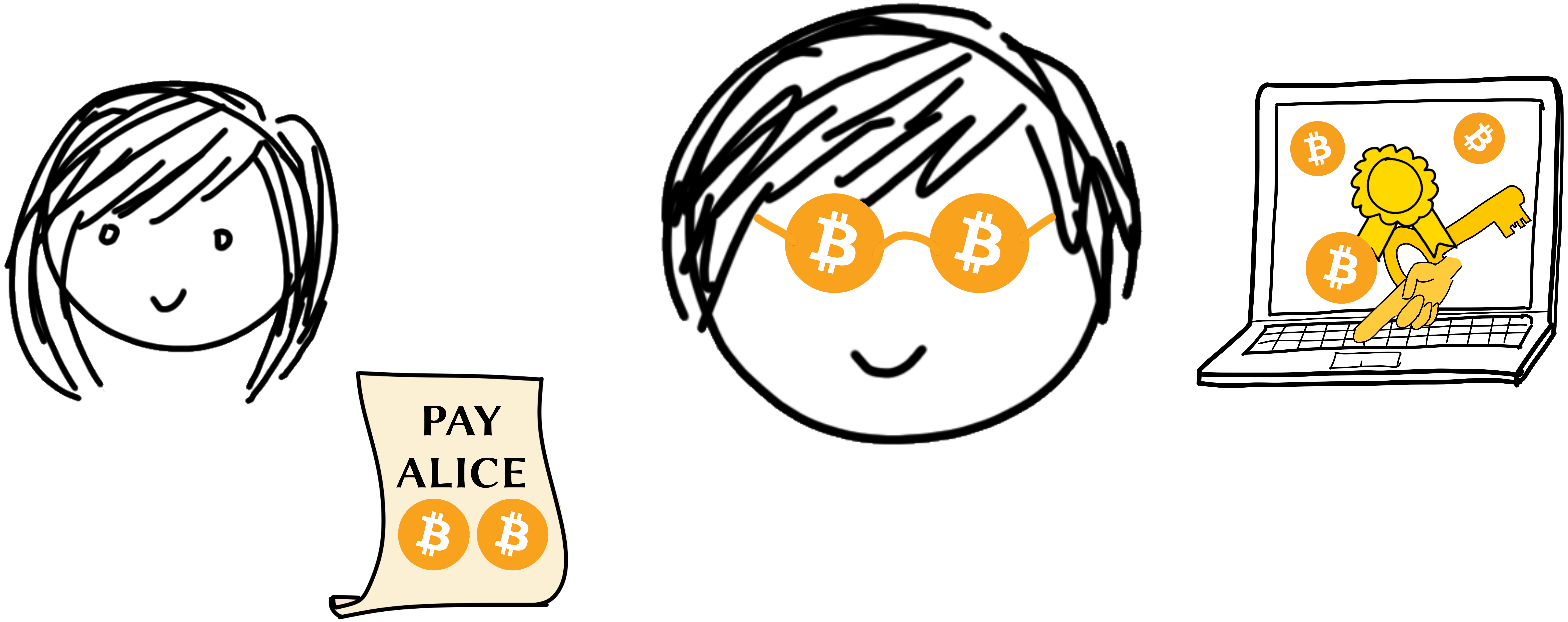

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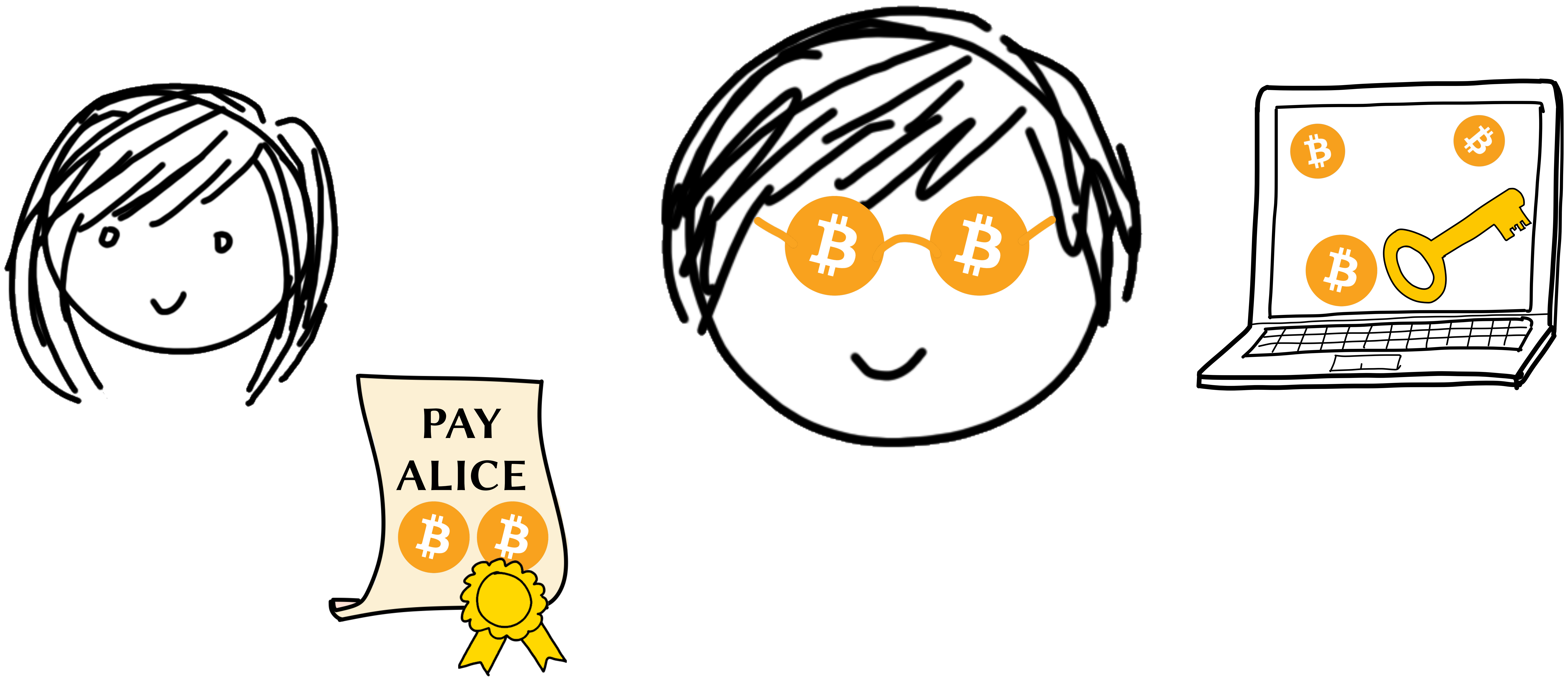

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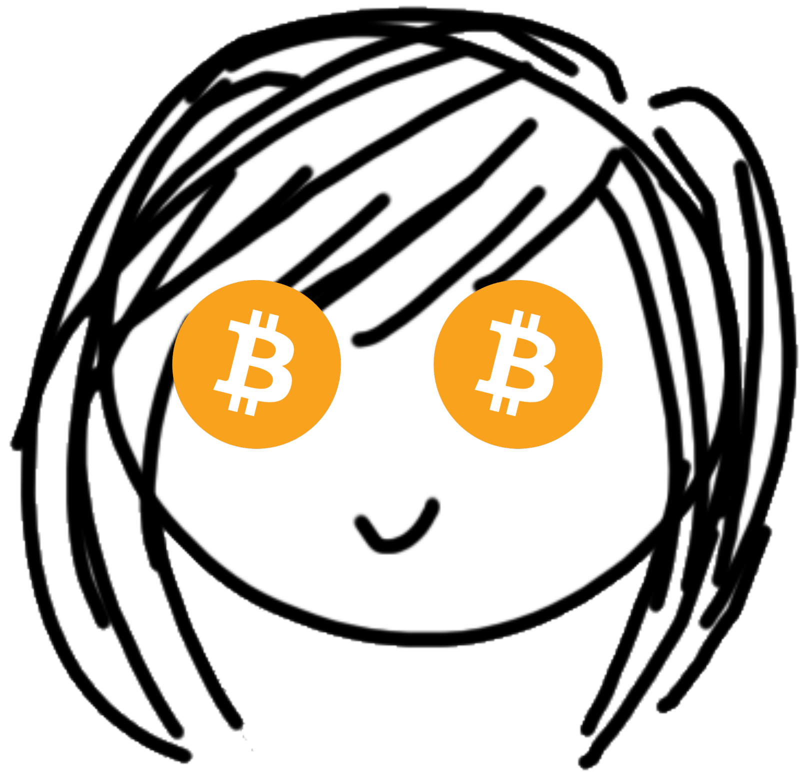

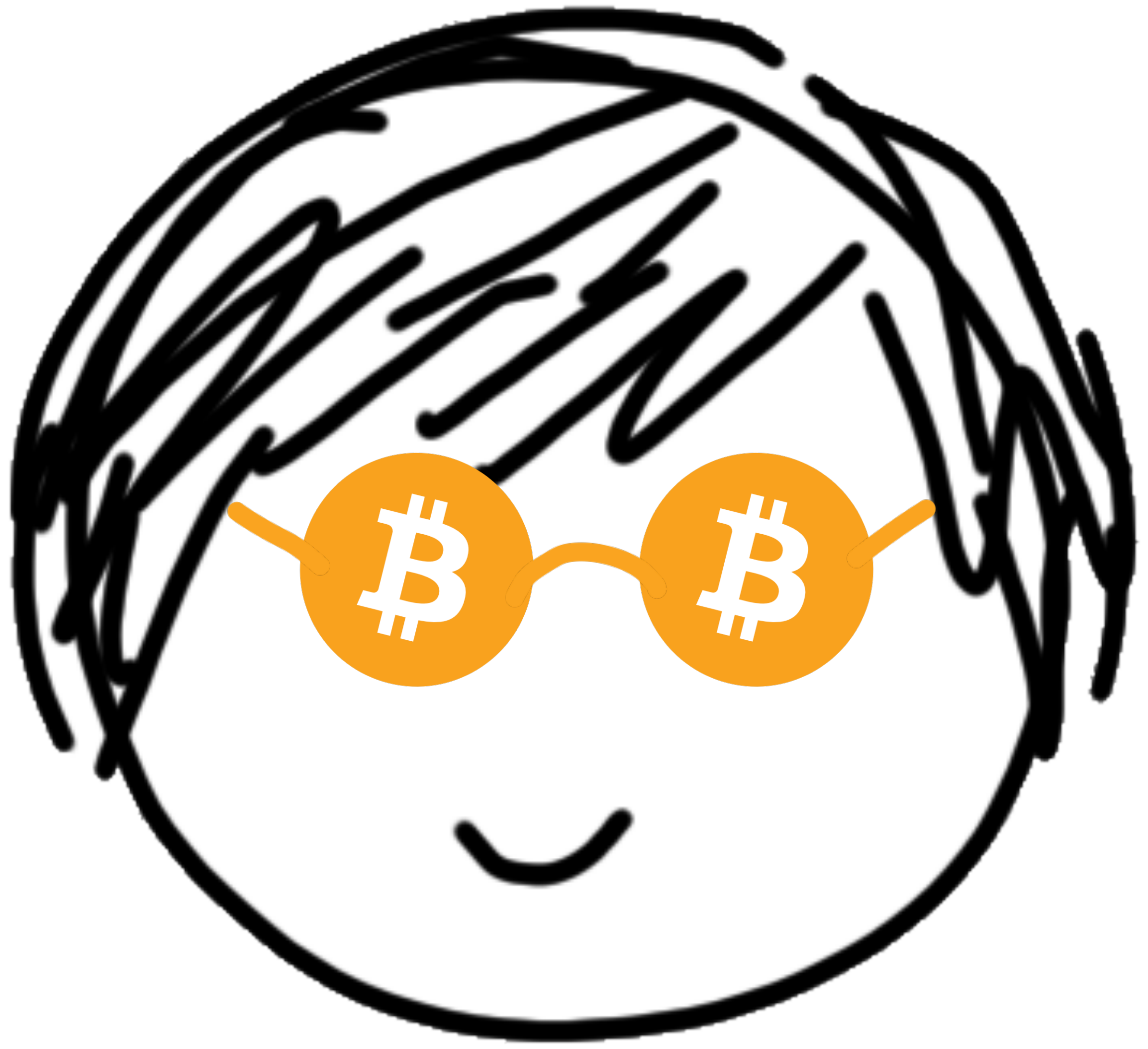

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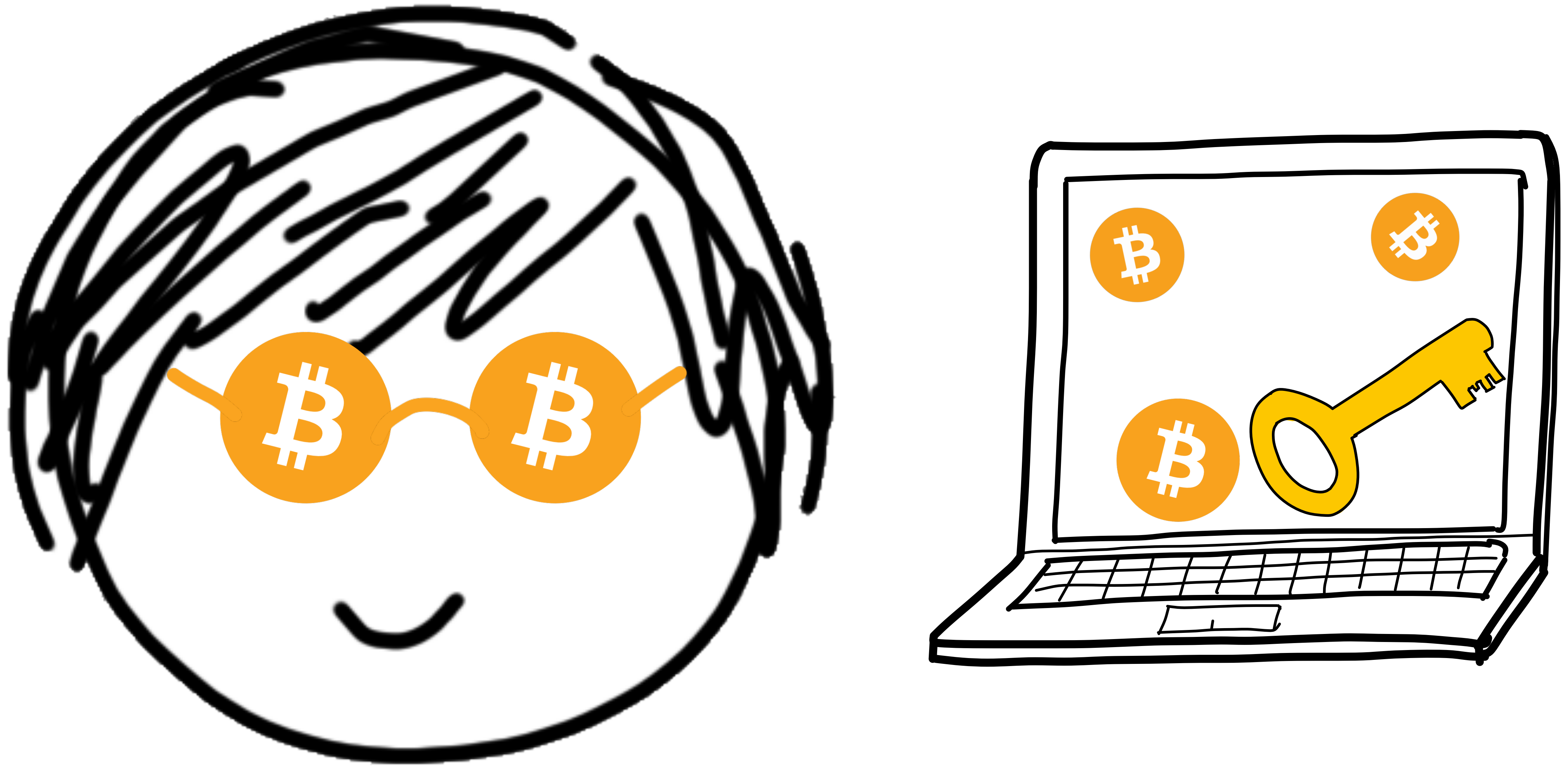

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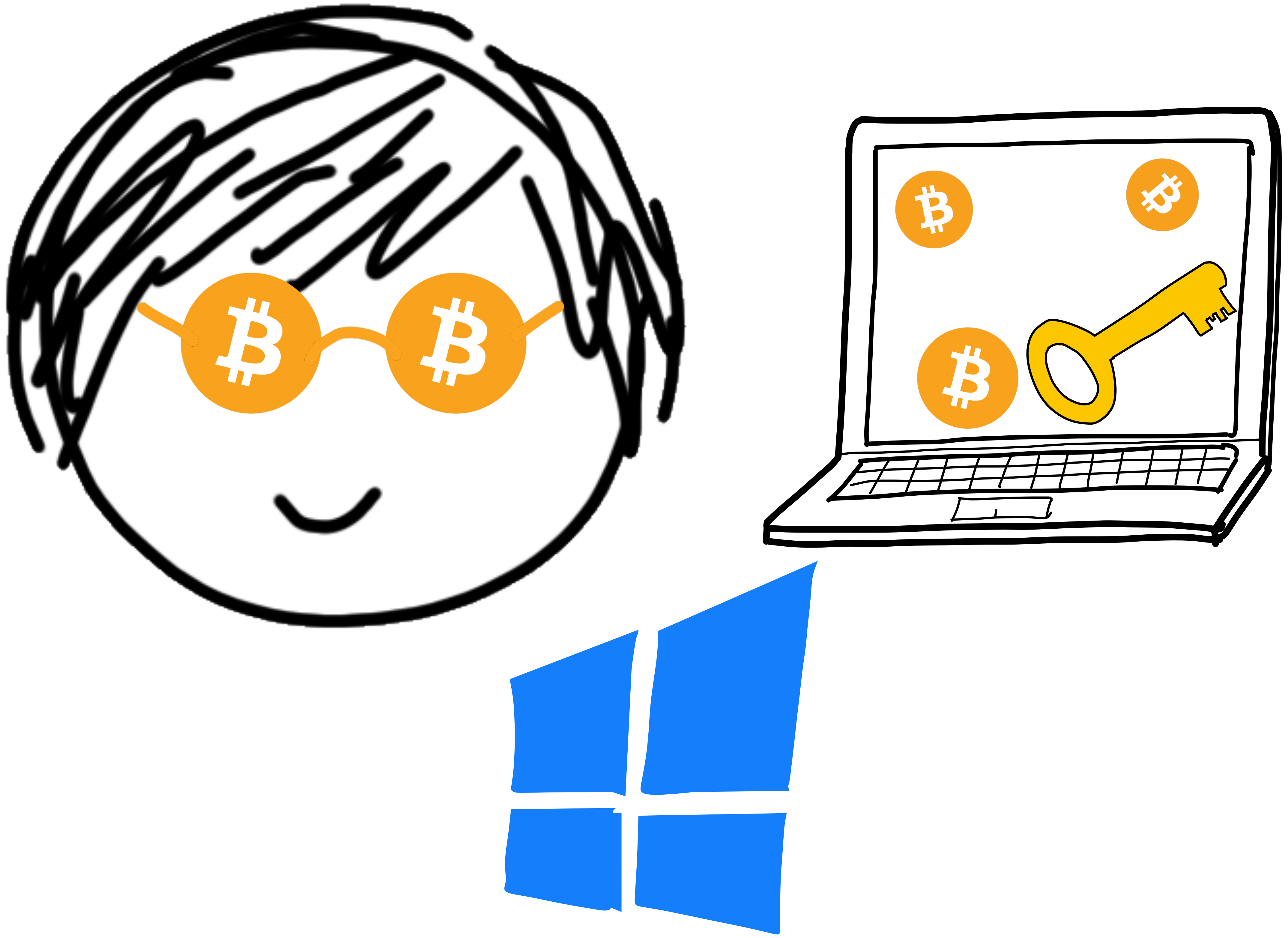

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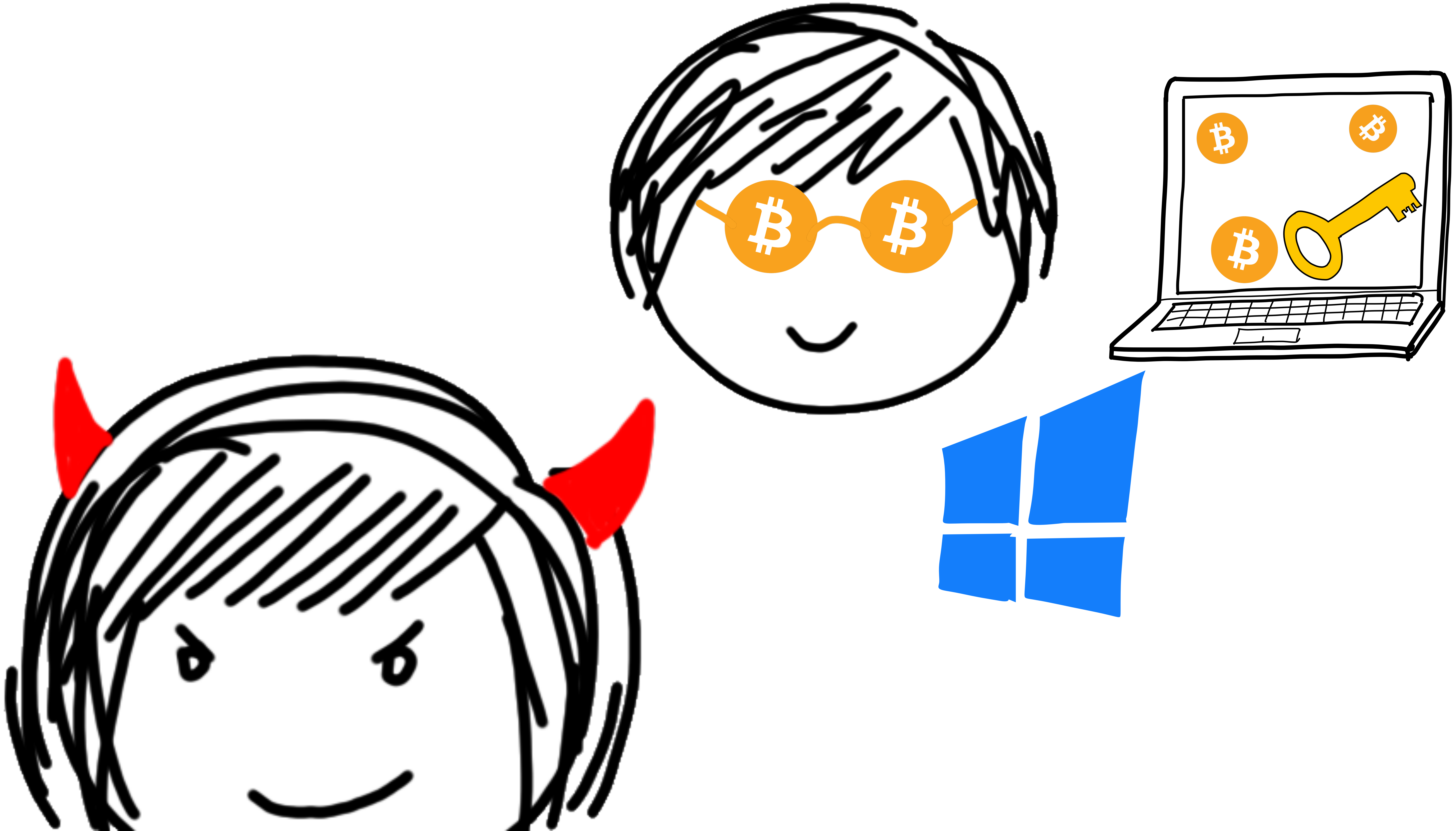

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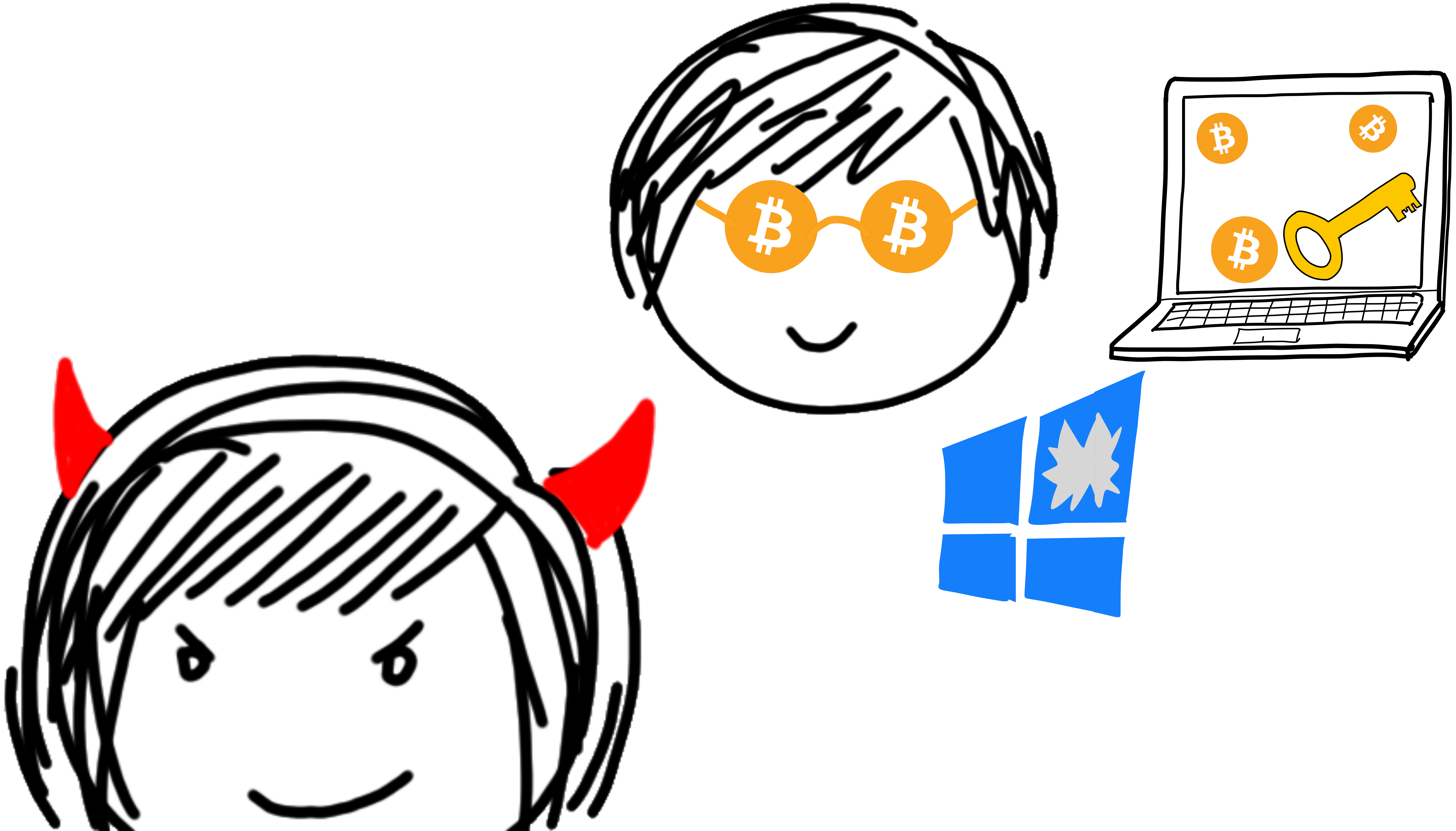

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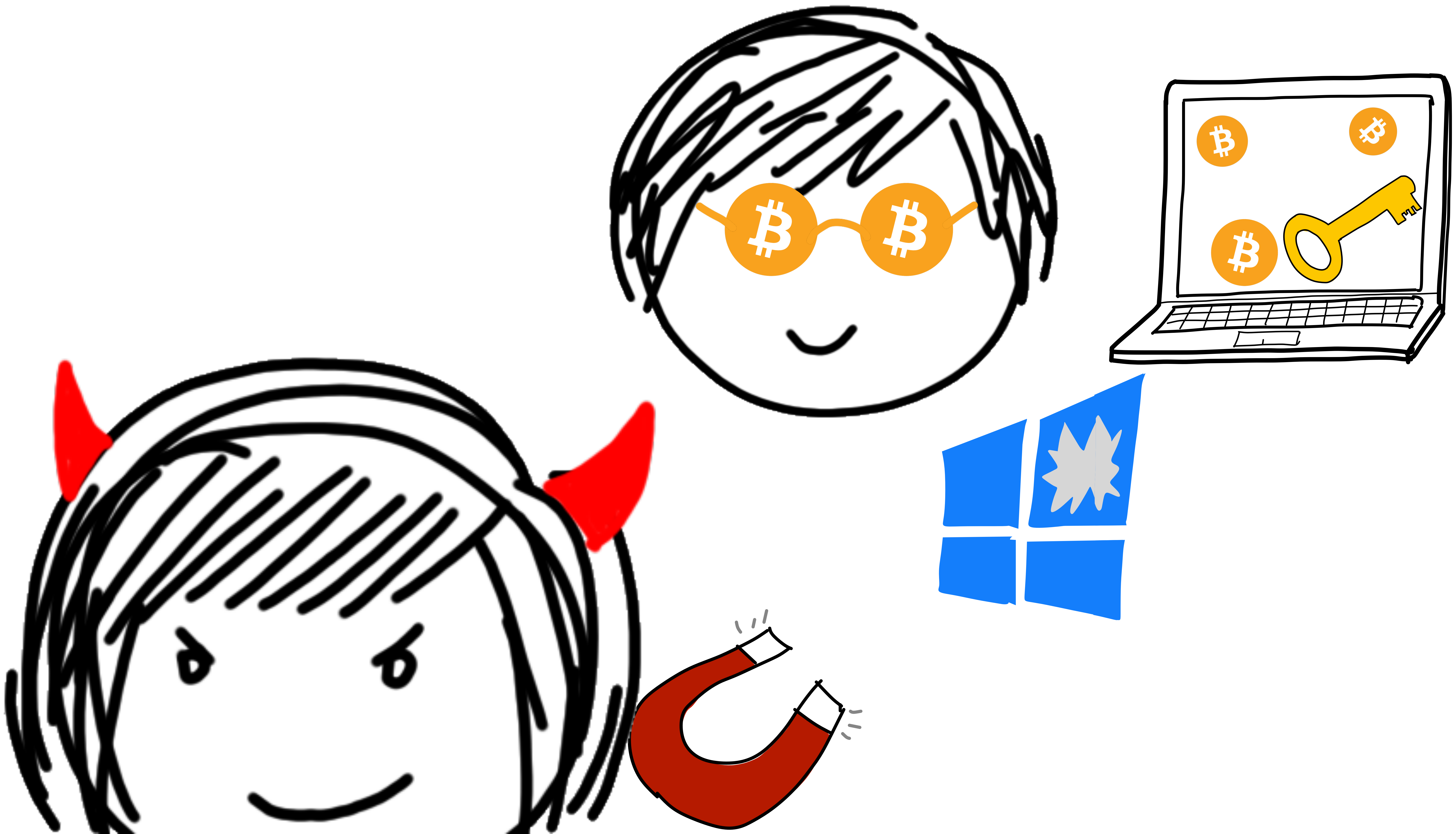

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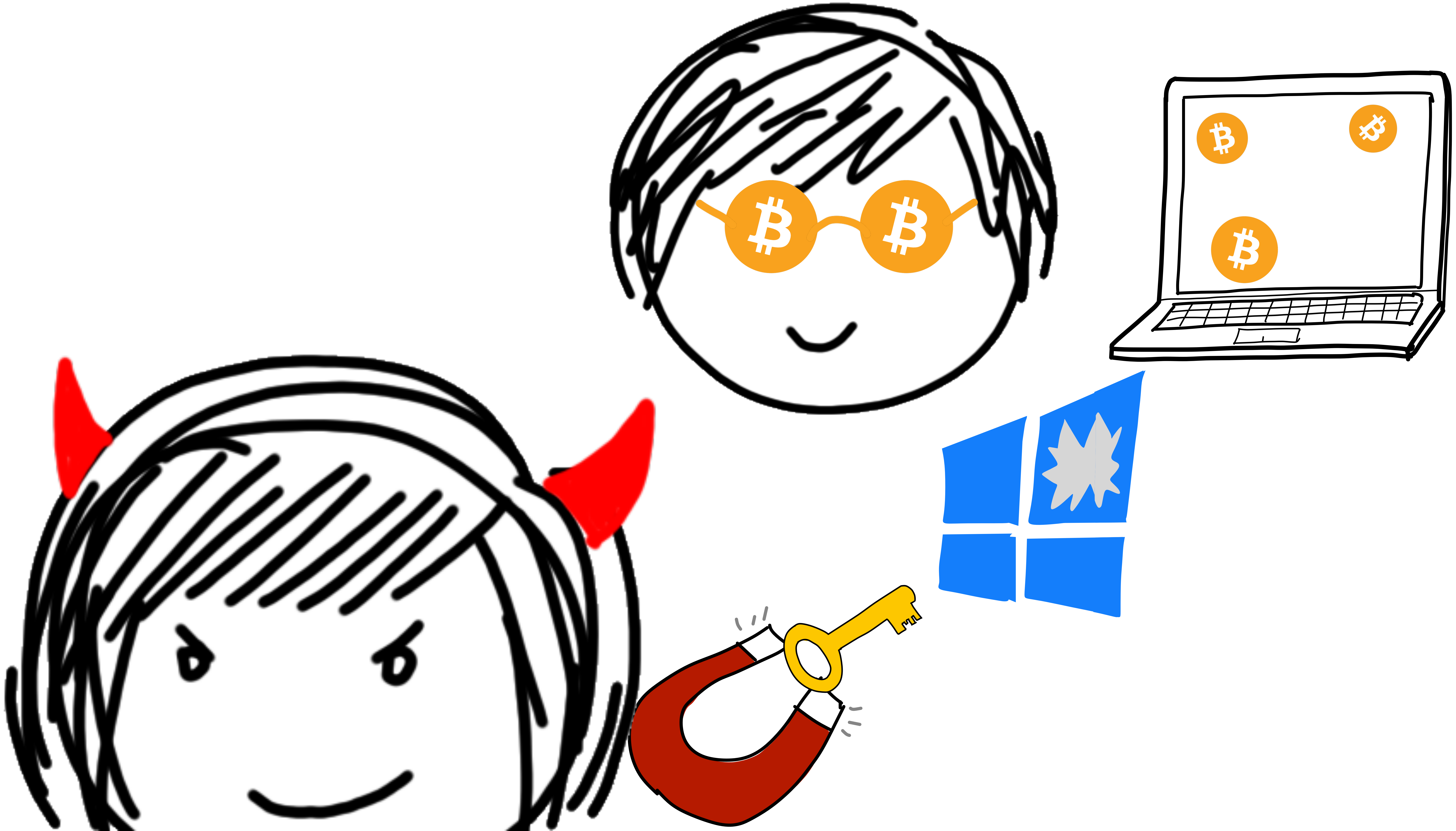

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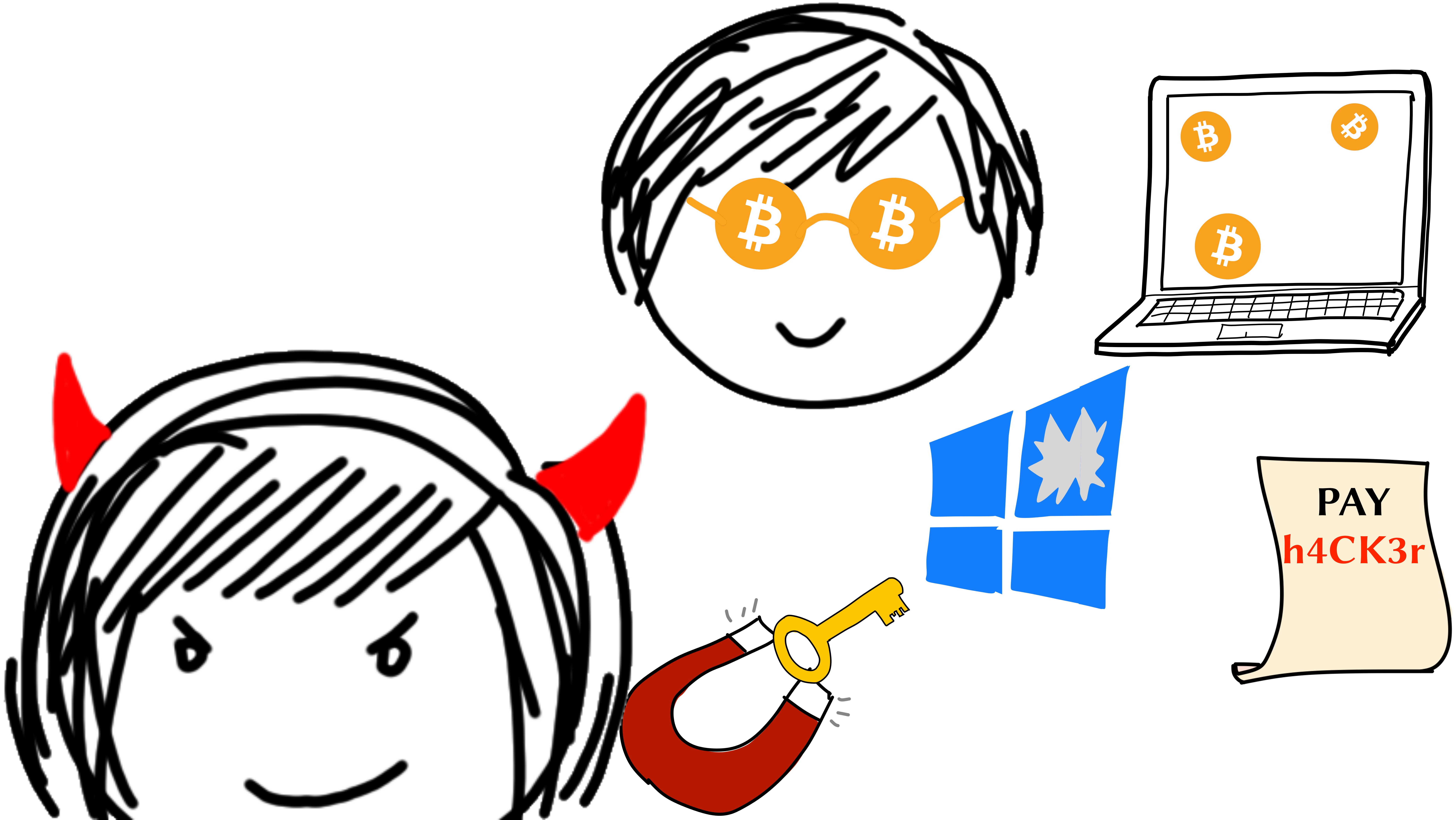

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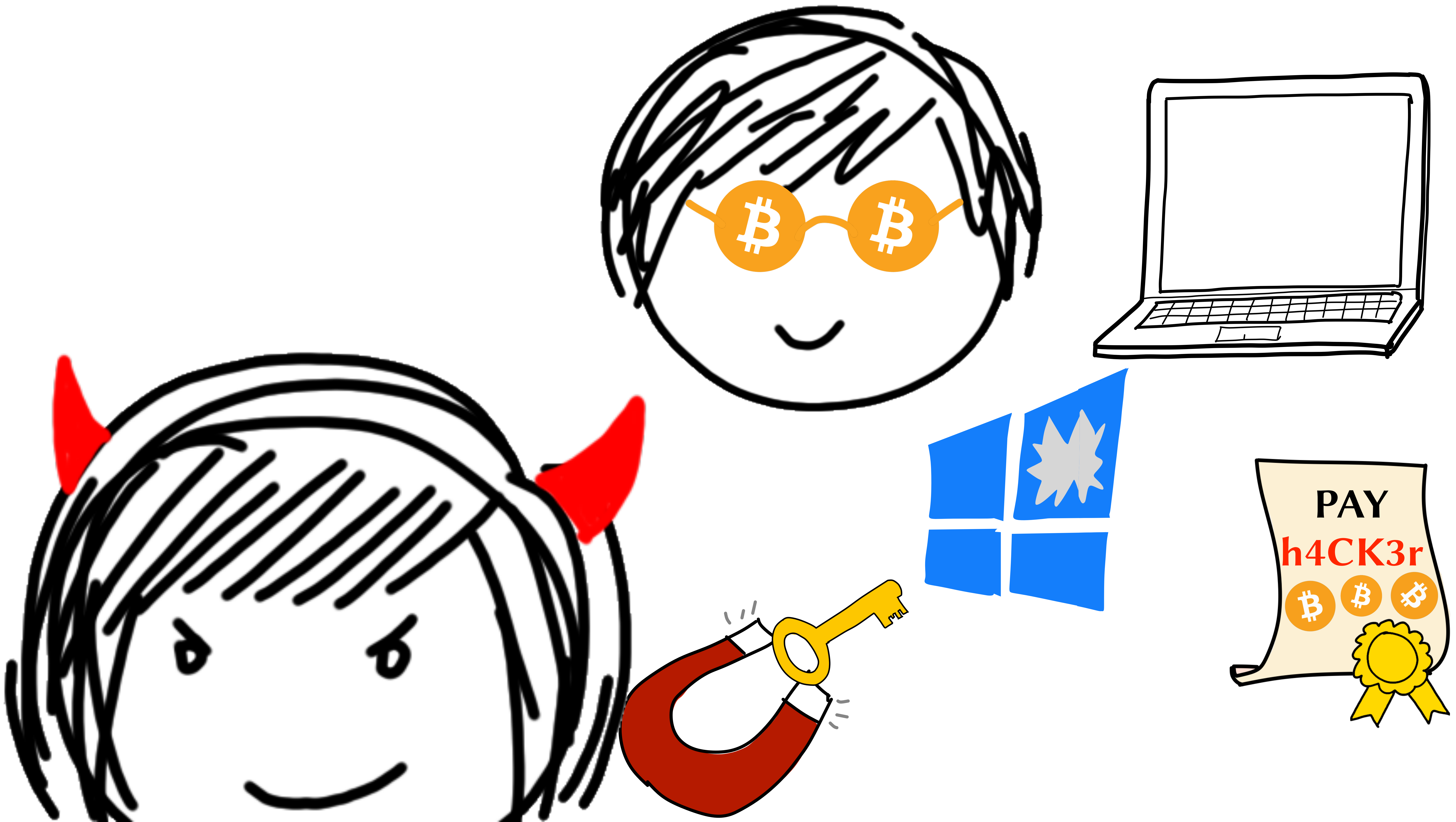

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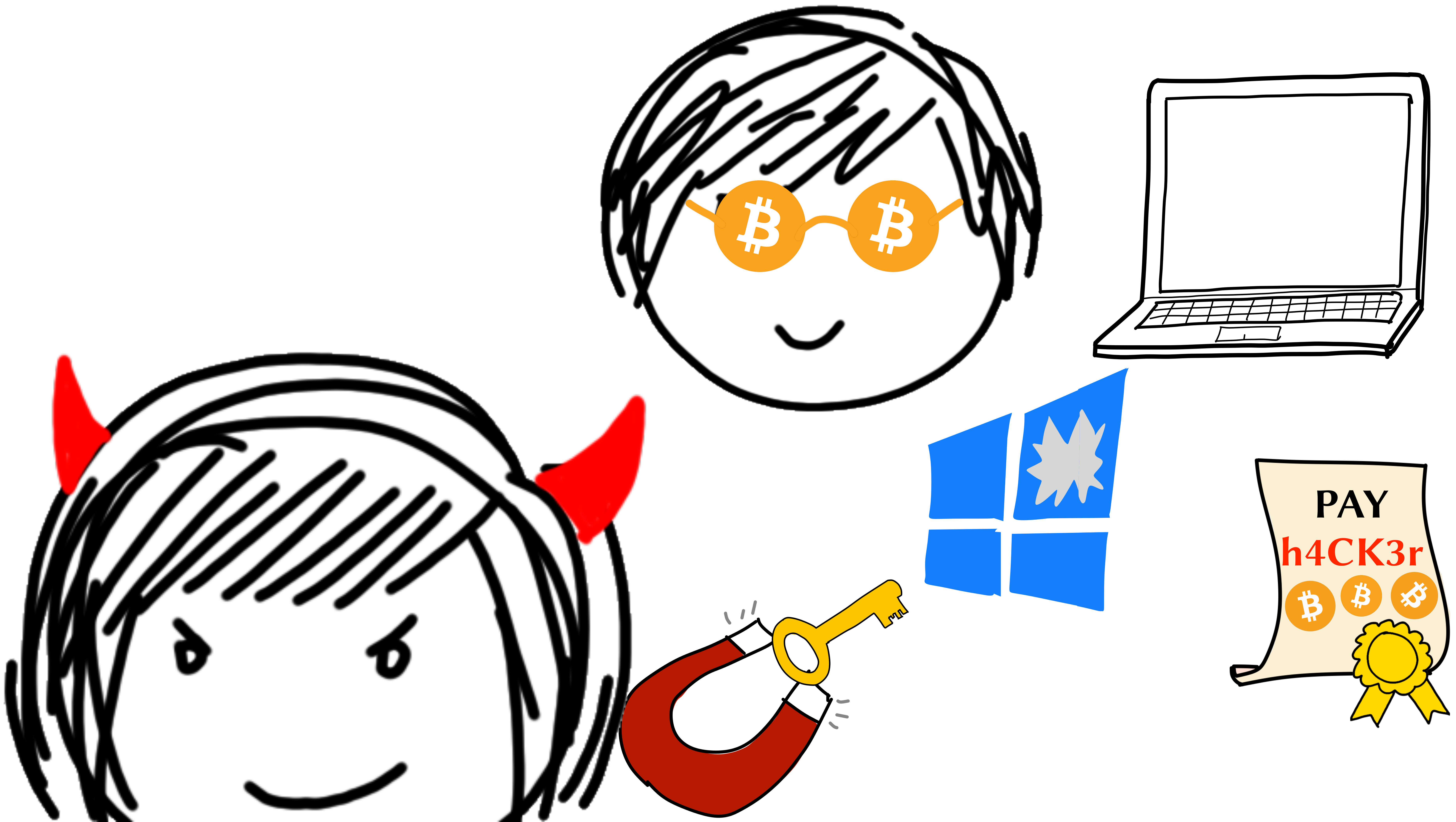

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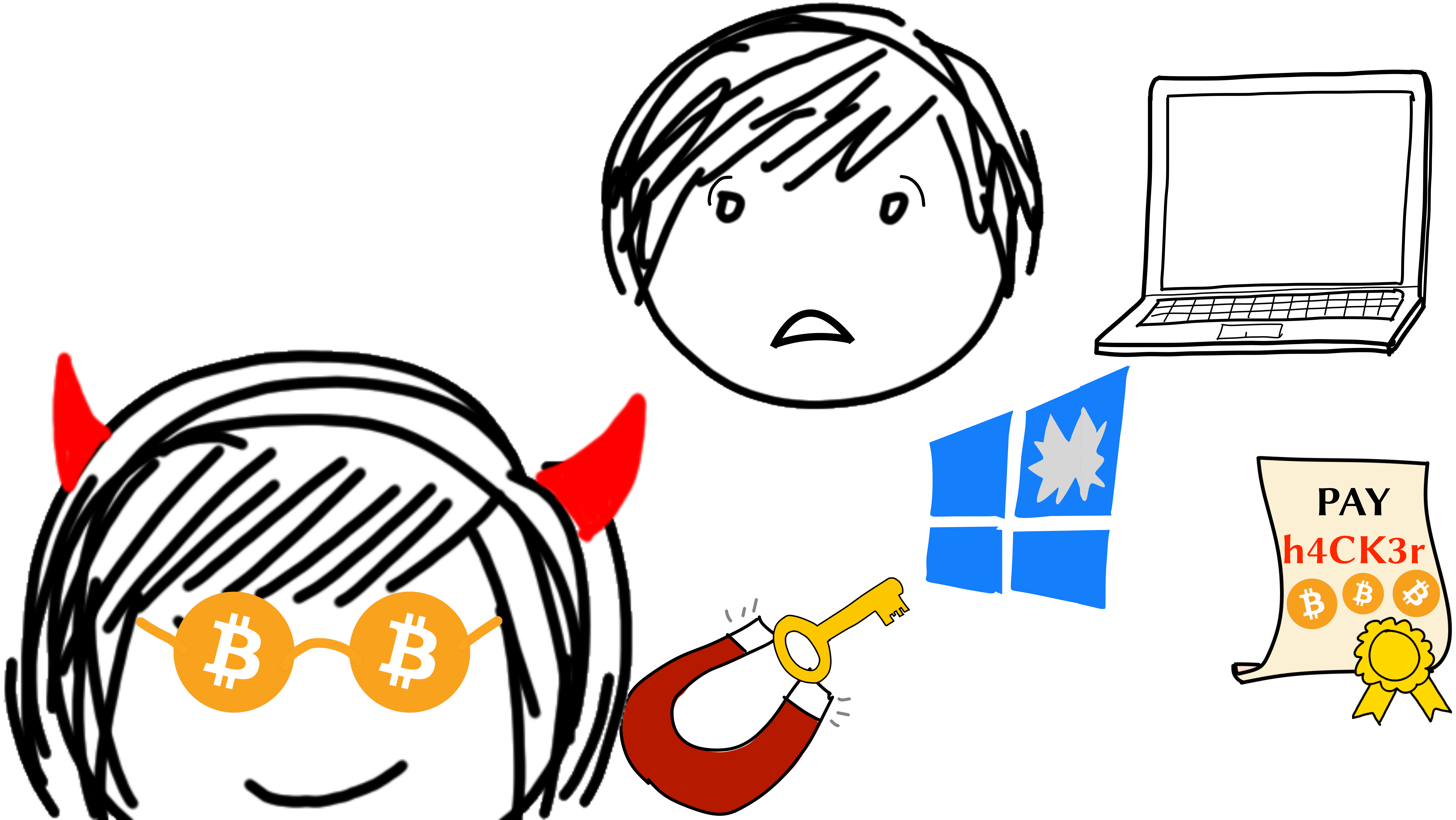

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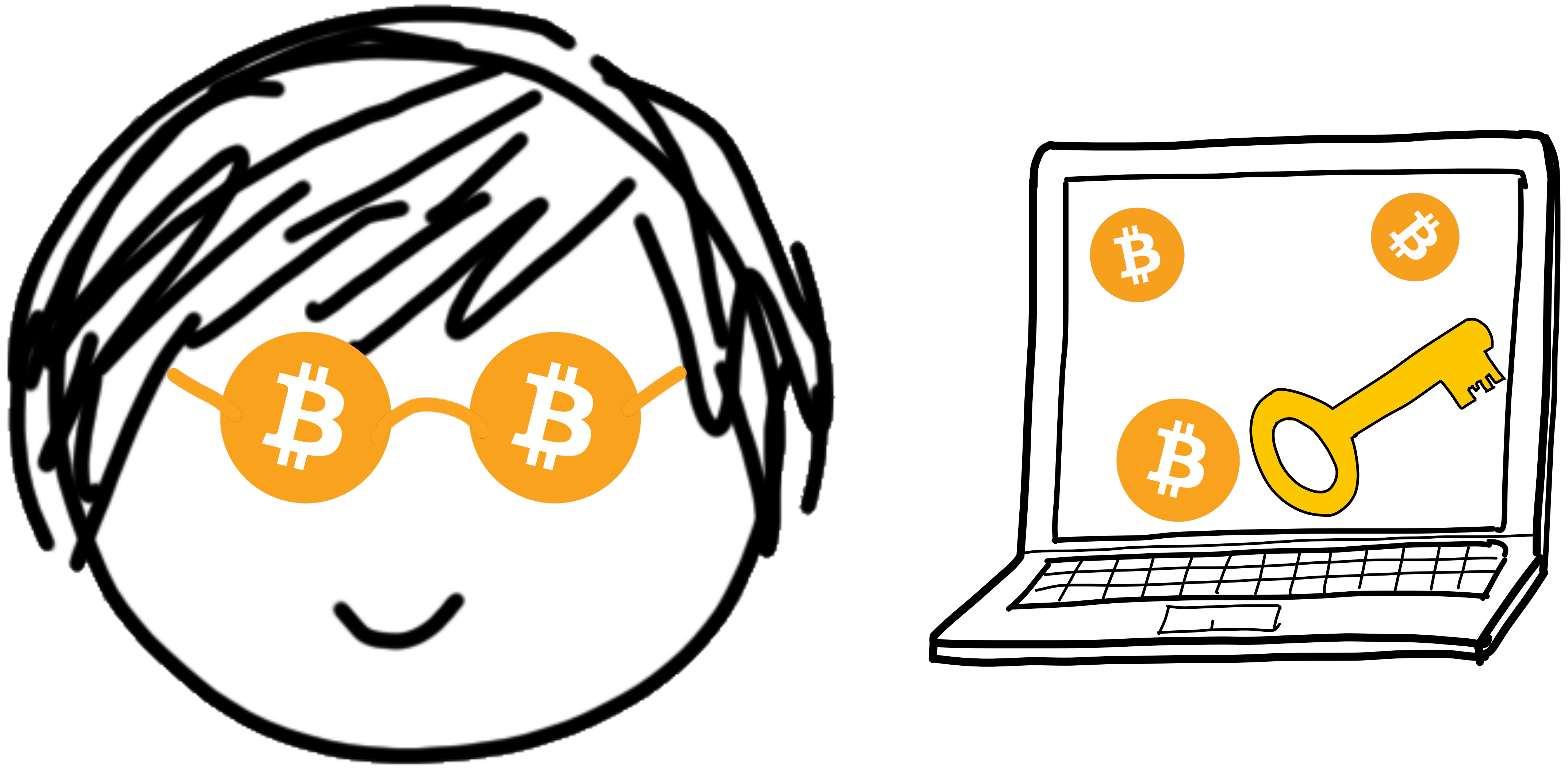

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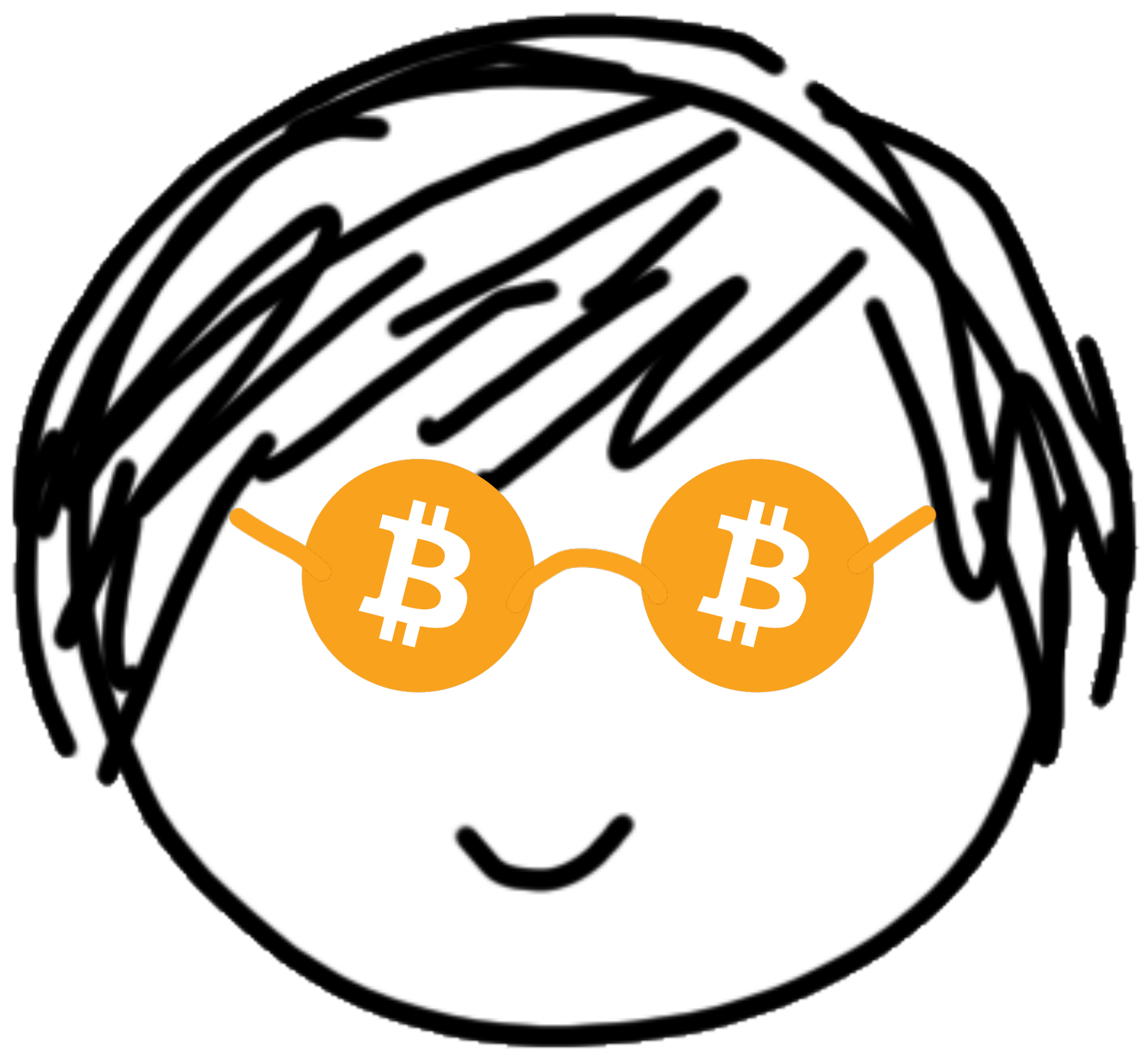

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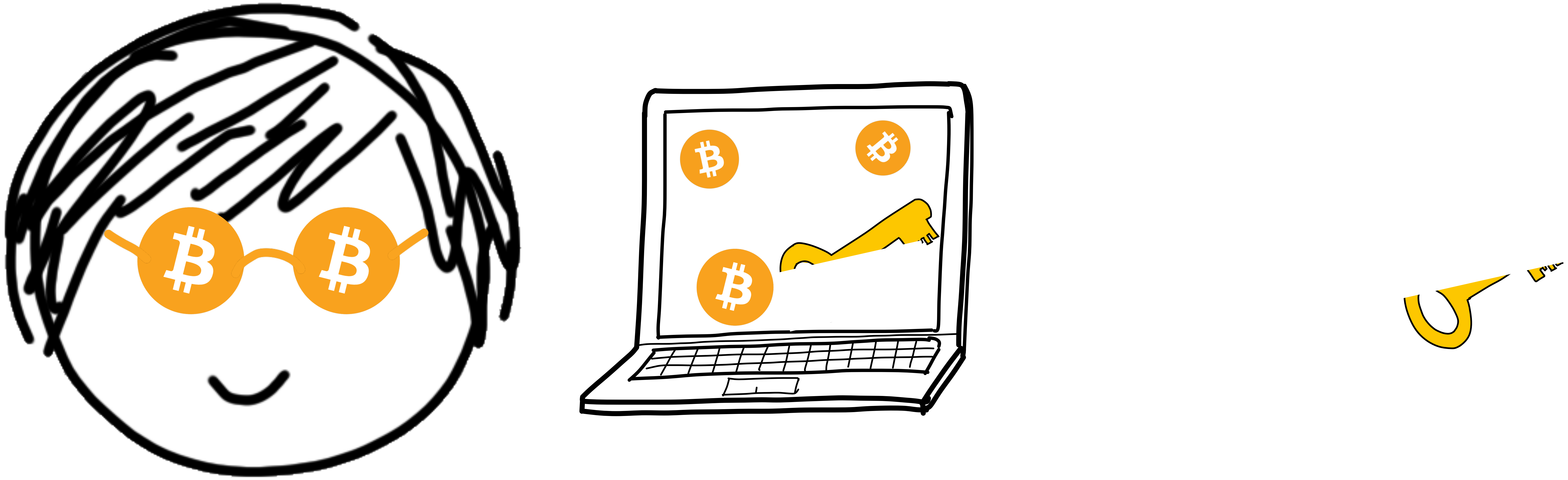

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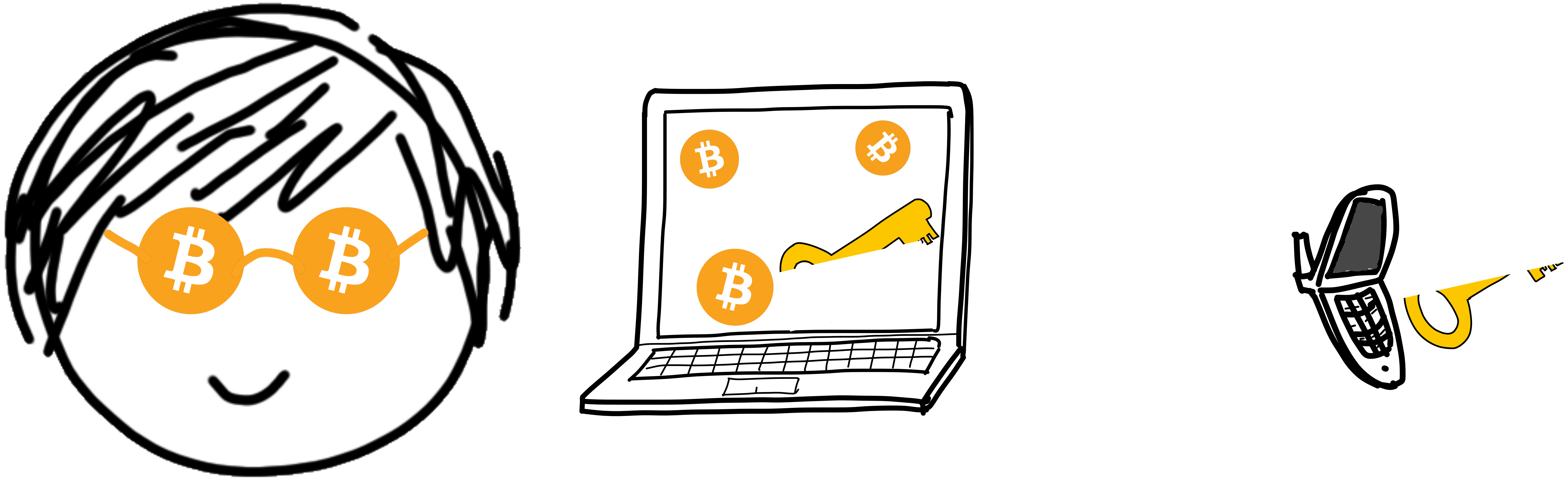

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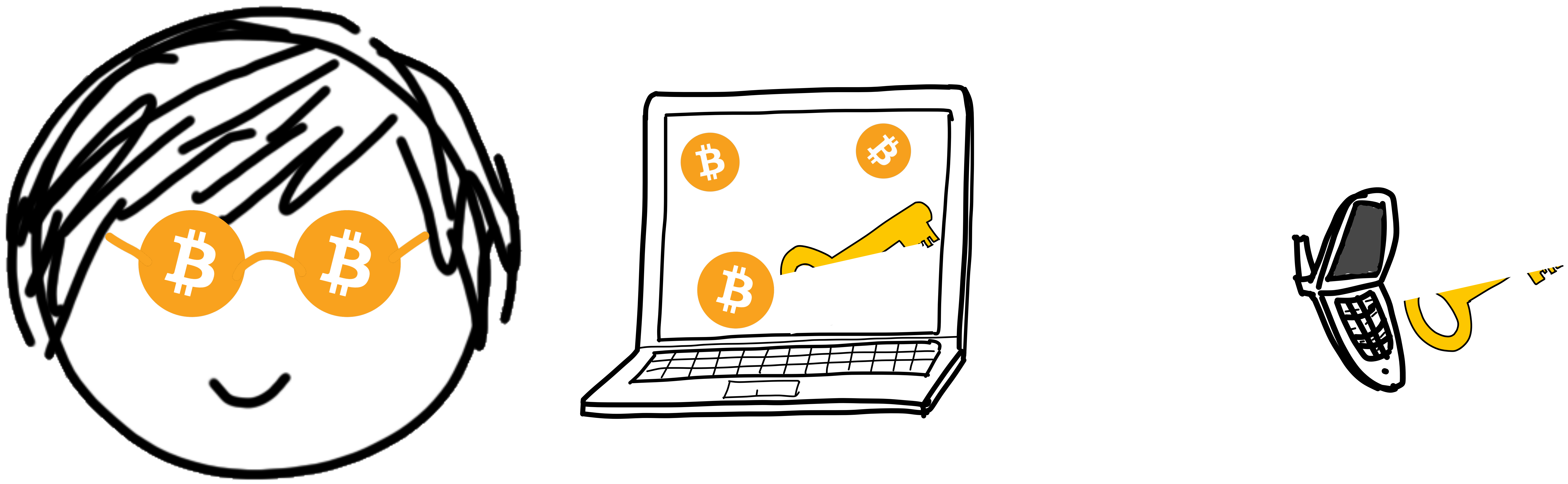

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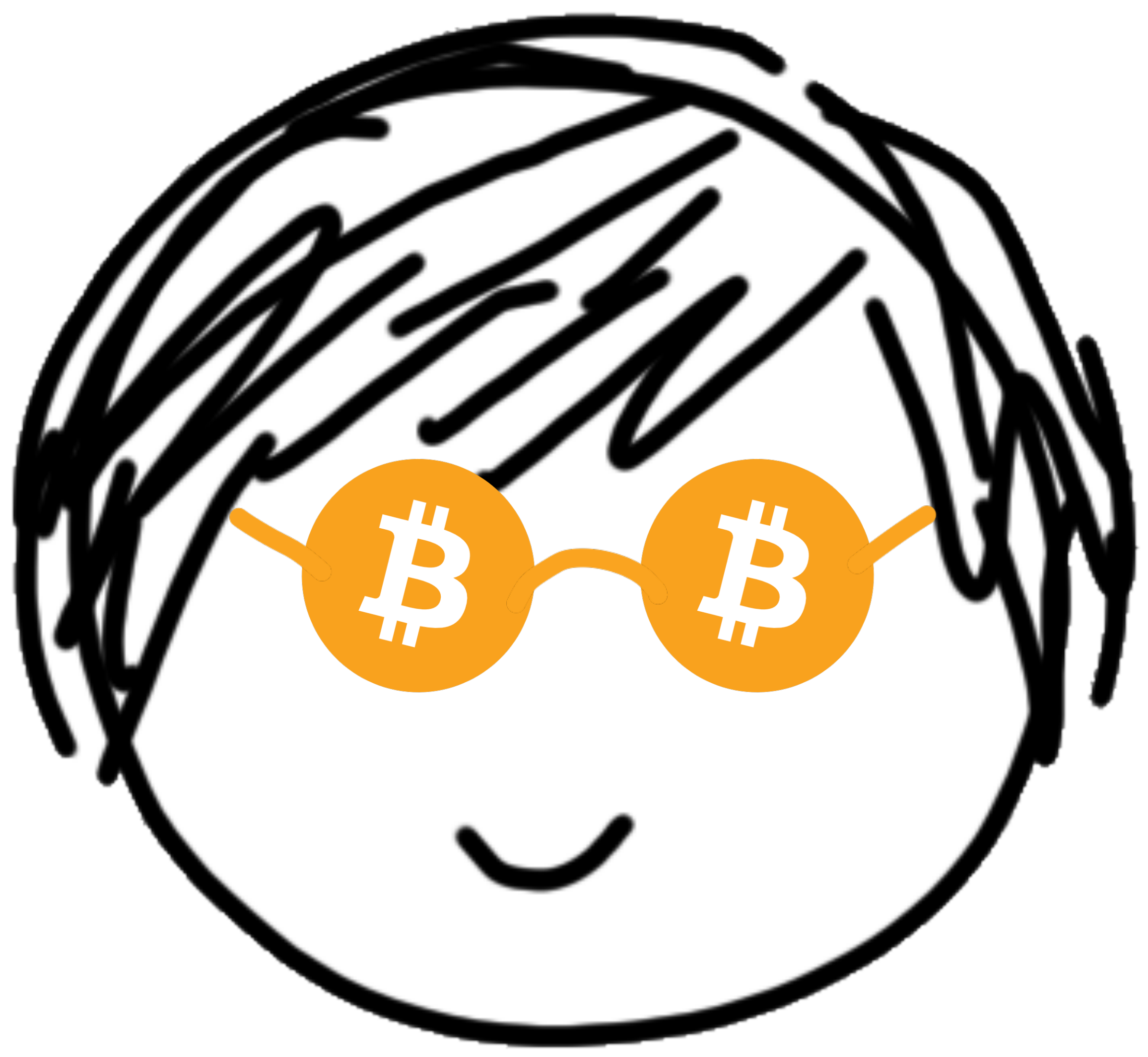

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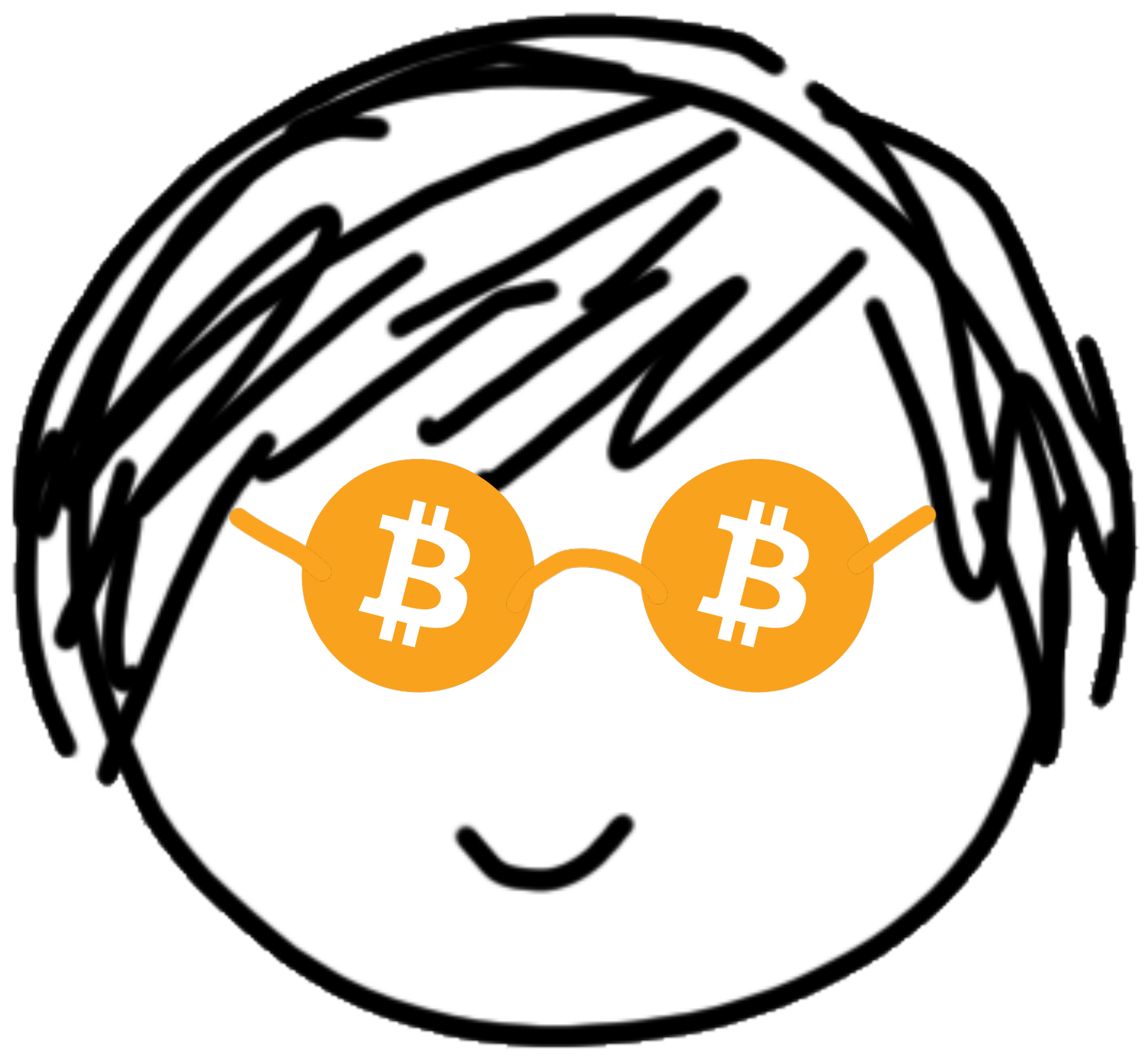

Ethereum Eysa

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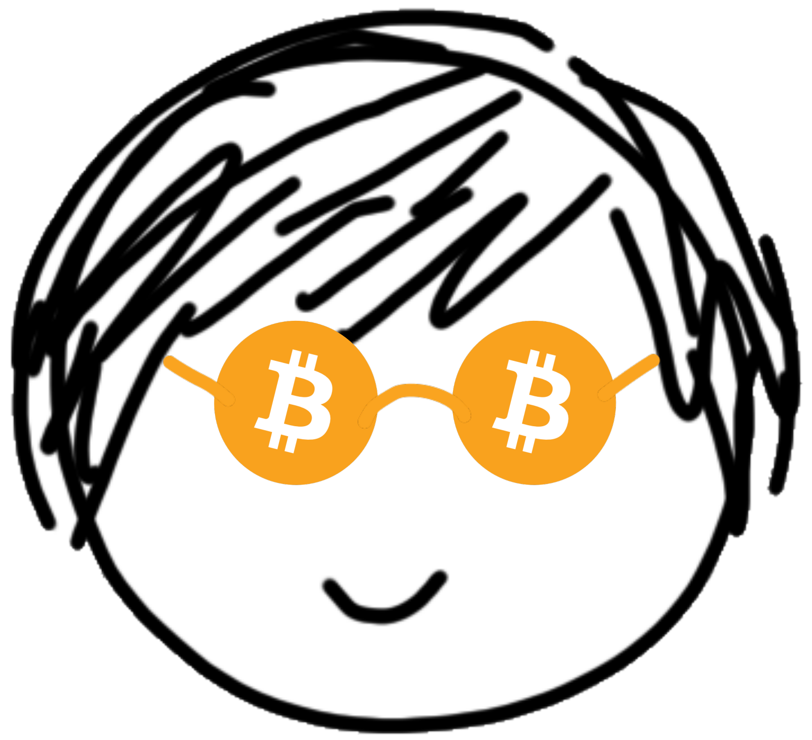

>$1T economy vulnerable to single points of failure in key management

Dogecoin Doerner

Ethereum Eysa

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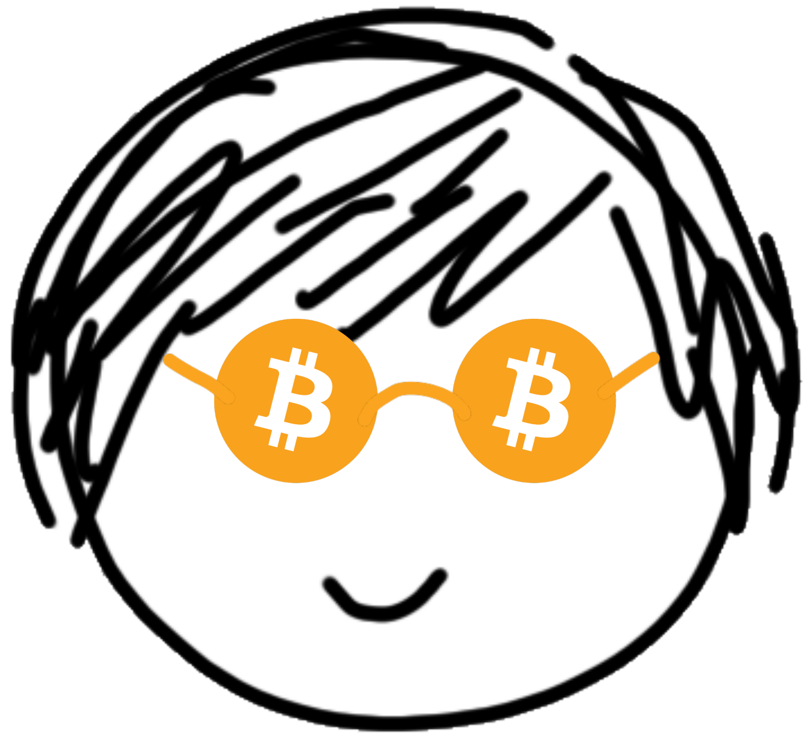

Large?

~~>$1T~~ **economy** vulnerable to single points of failure in key management

Dogecoin Doerner

Intro

How to distribute ECDSA

Tradeoffs

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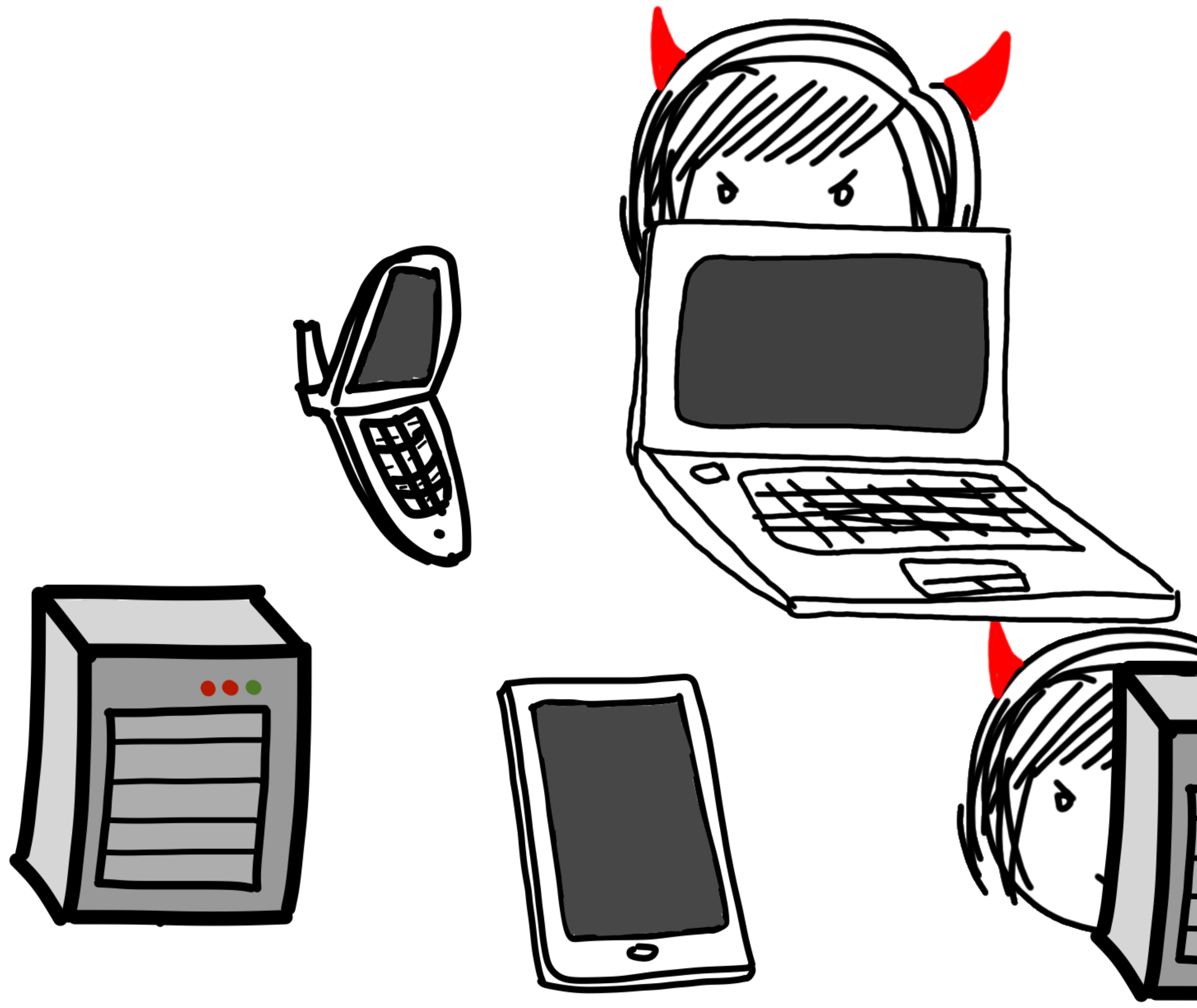
Tradeoffs

MPC for Schnorr is easy

but not ECDSA

Adversary Model

- Corruption threshold



Dishonest majority
(only one device uncompromised)

Adversary Model

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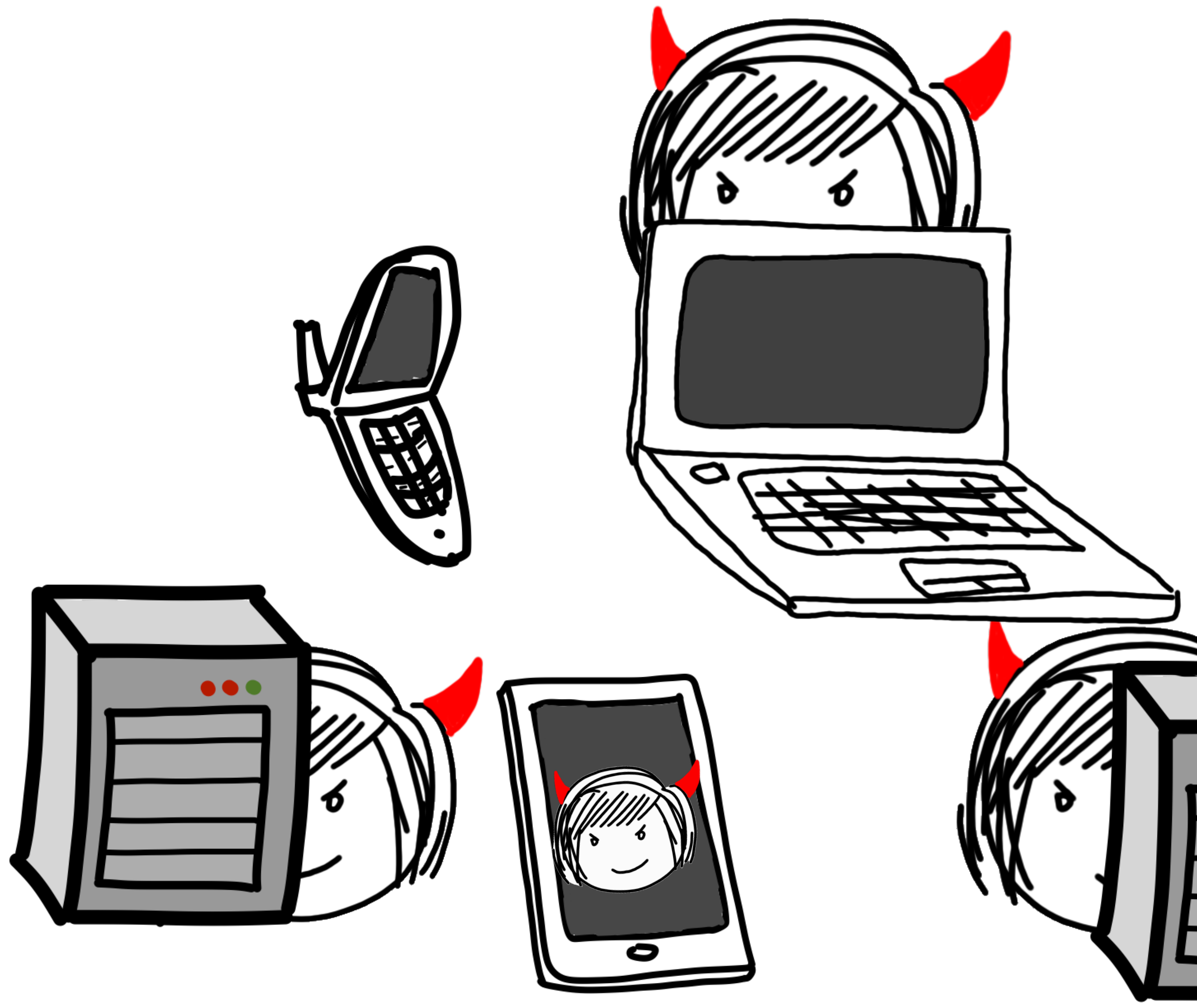

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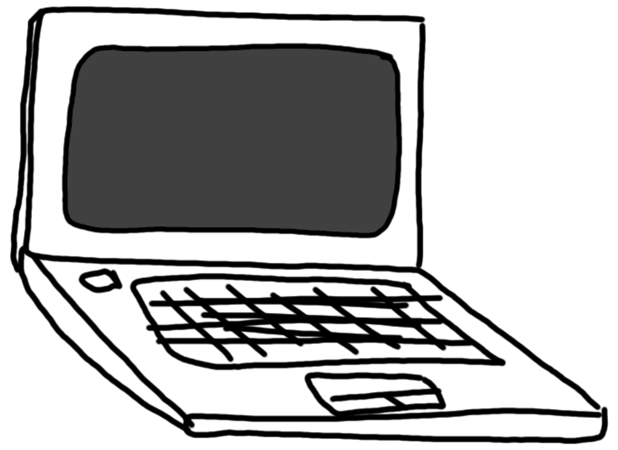

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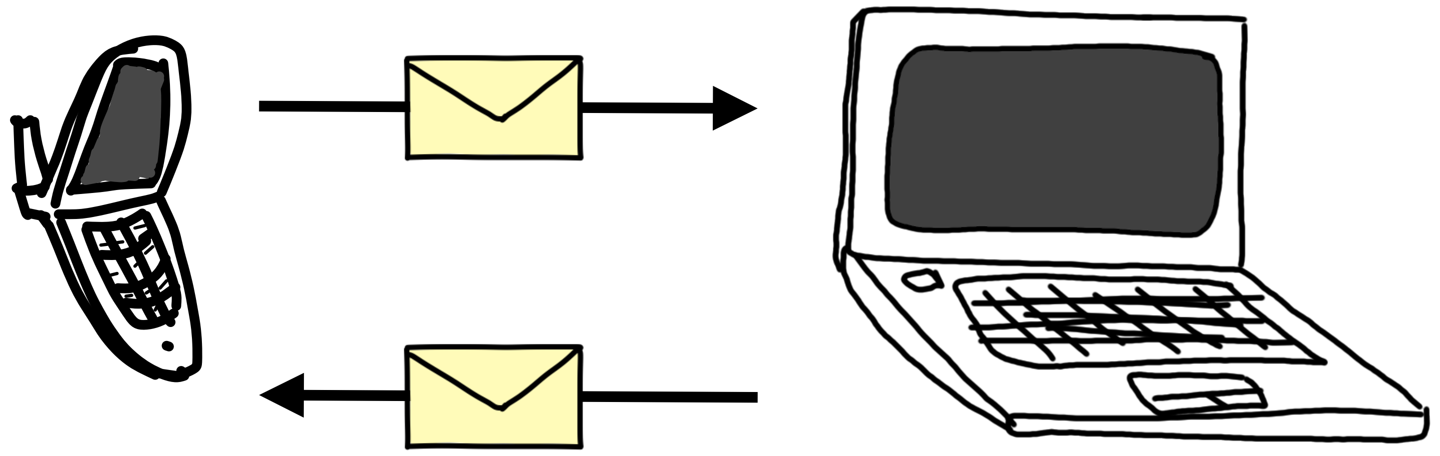

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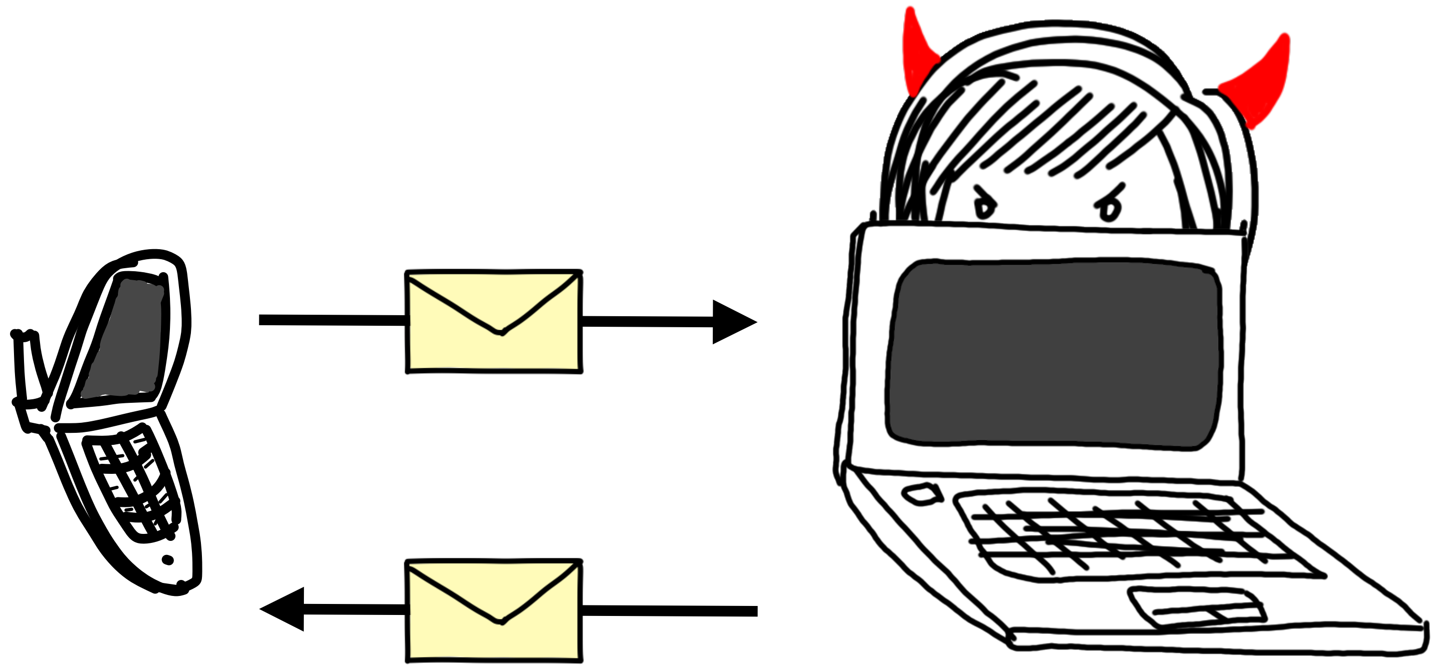

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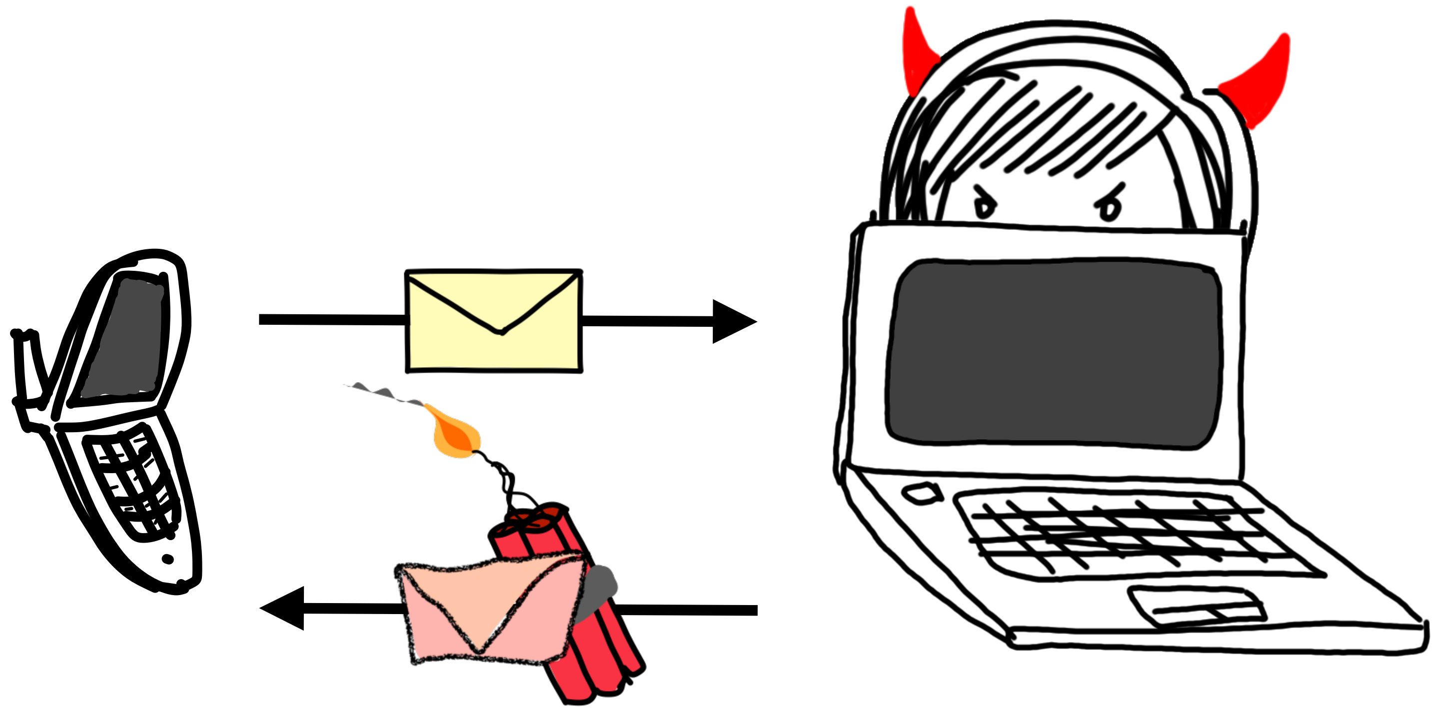

Malicious
(arbitrary deviations from protocol)

Concrete Example: Schnorr Signatures

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- Elegant signature scheme based on the Discrete Logarithm problem [Schnorr 89]

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For certain elliptic curves, best known algorithms for DLP run in time $\Theta\left(\sqrt{q}\right)$

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Very informally: $x \to X$ ***EASY***

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$x \rightarrow X$ ***EASY***

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Very informally:

$x \to X$	*EASY*	$30\mu s$
$X \to x$	HARD	Many billion billions of years

Schnorr Key Generation

$$\mathrm{SchnorrKeyGen}(\mathbb{G}, G, q):$$
$$\mathsf{sk} \leftarrow \mathbb{Z}_q$$
$$\mathsf{PK} = \mathsf{sk} \cdot G$$
$$\text{output } (\mathsf{sk}, \mathsf{PK})$$

<u>s</u>ecret <u>k</u>ey: kept private

<u>P</u>ublic <u>K</u>ey: exposed to the outside world

Schnorr Signing

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$$\text{output } (\mathsf{sk}, \mathsf{PK})$$

$\mathsf{SchnorrSign}(\mathsf{sk}, m):$

$$k \leftarrow \mathbb{Z}_q$$

$$\boxed{R} = k \cdot G$$

NONCE
One-time use value

Schnorr Signing

$\mathrm{SchnorrKeyGen}(\mathbb{G}, G, q):$

$$\mathsf{sk} \leftarrow \mathbb{Z}_q$$

$$\mathsf{PK} = \mathsf{sk} \cdot G$$

$$\text{output } (\mathsf{sk}, \mathsf{PK})$$

$\mathrm{SchnorrSign}(\mathsf{sk}, m):$

$$k \leftarrow \mathbb{Z}_q$$

$$\boxed{R} = k \cdot G$$

NONCE
One-time use
value

$$e = H(R \| m)$$

Schnorr Signing

$$\mathrm{SchnorrKeyGen}(\mathbb{G}, G, q):$$
$$\mathsf{sk} \leftarrow \mathbb{Z}_q$$
$$\mathsf{PK} = \mathsf{sk} \cdot G$$
$$\text{output } (\mathsf{sk}, \mathsf{PK})$$

$$\mathrm{SchnorrSign}(\mathsf{sk}, m):$$
$$k \leftarrow \mathbb{Z}_q$$
$$\boxed{R} = k \cdot G$$
$$e = H(R \| m)$$
$$s = k - \mathsf{sk} \cdot e \pmod q$$

NONCE
One-time use value

Schnorr Signing

$\text{SchnorrKeyGen}(\mathbb{G}, G, q):$

$$\mathsf{sk} \leftarrow \mathbb{Z}_q$$

$$\mathsf{PK} = \mathsf{sk} \cdot G$$

$$\text{output } (\mathsf{sk}, \mathsf{PK})$$

$\text{SchnorrSign}(\mathsf{sk}, m):$

$$k \leftarrow \mathbb{Z}_q$$

$$\boxed{R} = k \cdot G$$

NONCE
One-time use value

$$e = H(R \| m)$$

$$s = k - \mathsf{sk} \cdot e \pmod{q}$$

$$\sigma = (s, R)$$

$$\text{output } \sigma$$

Schnorr Signing

$\text{SchnorrKeyGen}(\mathbb{G}, G, q):$

$$\mathsf{sk} \leftarrow \mathbb{Z}_q$$

$$\mathsf{PK} = \mathsf{sk} \cdot G$$

$$\text{output } (\mathsf{sk}, \mathsf{PK})$$

$\text{SchnorrSign}(\mathsf{sk}, m):$

$$k \leftarrow \mathbb{Z}_q$$

$$\boxed{R} = k \cdot G$$

NONCE
One-time use value

$$e = H(R \| m)$$

$$\boxed{s = k - \mathsf{sk} \cdot e} \pmod q$$

$$\sigma = (s, R)$$

$$\text{output } \sigma$$

Verifying a signature: $\boxed{s \cdot G \stackrel{?}{=} R - e \cdot \mathsf{PK}}$

Schnorr Signing

$\text{SchnorrKeyGen}(\mathbb{G}, G, q):$

$\mathsf{sk} \leftarrow \mathbb{Z}_q$

$\mathsf{PK} = \mathsf{sk} \cdot G$

$\text{output } (\mathsf{sk}, \mathsf{PK})$

$\text{SchnorrSign}(\mathsf{sk}, m):$

$k \leftarrow \mathbb{Z}_q$

$\boxed{R} = k \cdot G$

NONCE
One-time use value

$e = H(R \| m)$

$\boxed{s = k - \mathsf{sk} \cdot e} \pmod q$

$\sigma = (s, R)$

$\text{output } \sigma$

Verifying a signature: $s \cdot G \stackrel{?}{=} \underbrace{\boxed{R}}_{k \cdot G} - e \cdot \underbrace{\boxed{\mathsf{PK}}}_{\mathsf{sk} \cdot G}$

Schnorr Signing

$\mathrm{SchnorrKeyGen}(\mathbb{G}, G, q):$

$$\mathsf{sk} \leftarrow \mathbb{Z}_q$$

$$\mathsf{PK} = \mathsf{sk} \cdot G$$

$$\text{output } (\mathsf{sk}, \mathsf{PK})$$

$\mathrm{SchnorrSign}(\mathsf{sk}, m):$

$$k \leftarrow \mathbb{Z}_q$$

$$R = k \cdot G$$

NONCE
One-time use value

$$e = H(R \| m)$$

$$s = k - \mathsf{sk} \cdot e \pmod{q}$$

$$\sigma = (s, R)$$

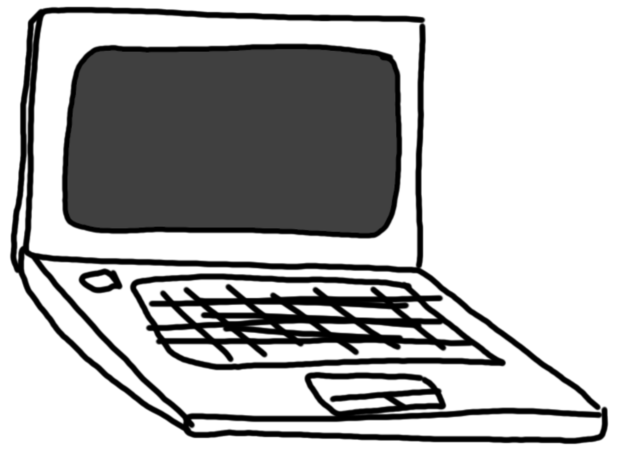
$$\text{output } \sigma$$

Verifying a signature: $s \cdot G \stackrel{?}{=} \boxed{R} - e \cdot \boxed{\mathsf{PK}}$

$$\boxed{k \cdot G} \qquad \boxed{\mathsf{sk} \cdot G}$$

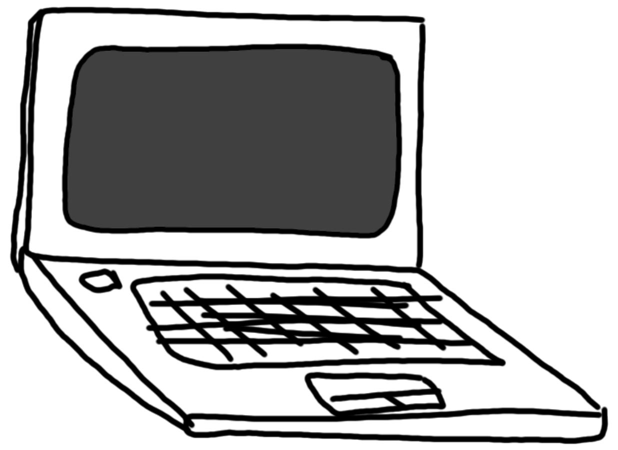
$$s \cdot G \stackrel{?}{=} (k - e \cdot \mathsf{sk}) \cdot G$$

Additive Secret Sharing



Additive Secret Sharing

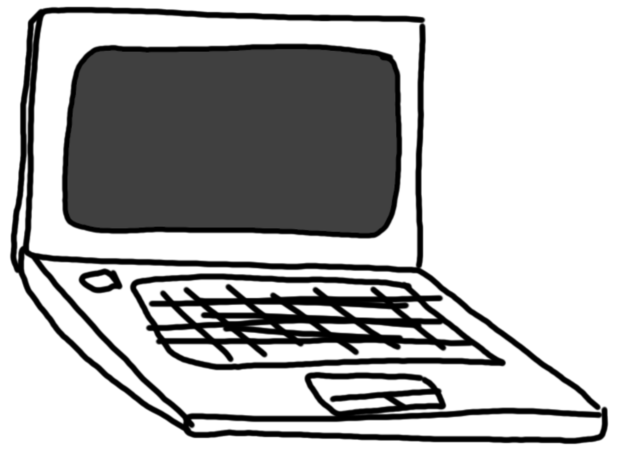
$$x \in \mathbb{Z}_q$$



Additive Secret Sharing

$$x \in \mathbb{Z}_q$$

$$x_A + x_B = x$$



Additive Secret Sharing

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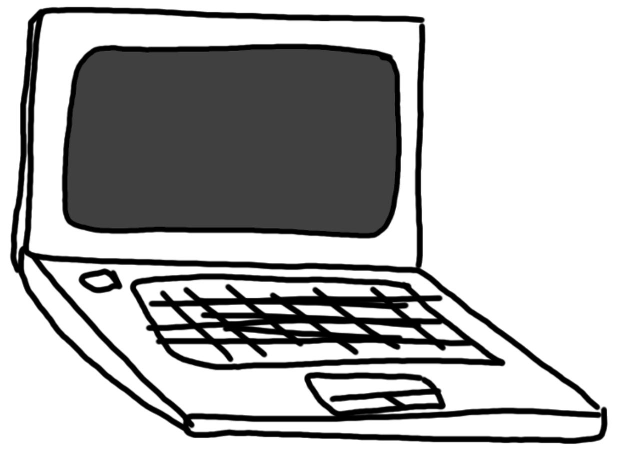


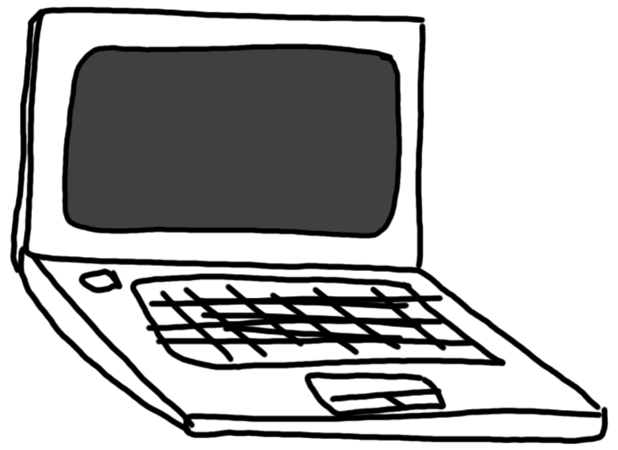
x_A

x_B

Additive Secret Sharing

$$x \in \mathbb{Z}_q$$

$$x_A + x_B = x$$

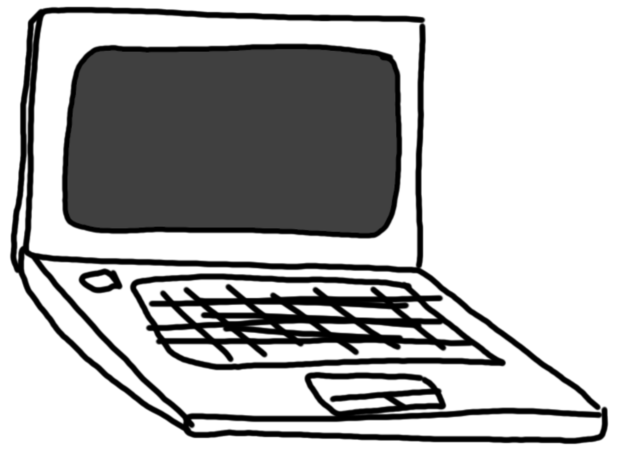


x_A $[x]$ x_B

Additive Secret Sharing

$$x \in \mathbb{Z}_q$$

$$x_A + x_B = x$$



x_A

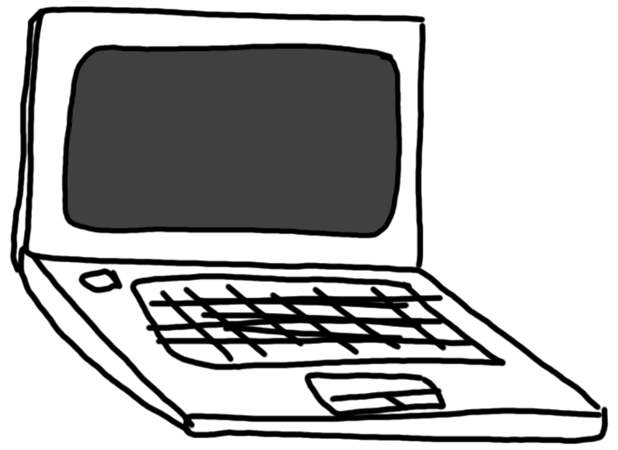
$[x]$

x_B

Additive Secret Sharing

$$x \in \mathbb{Z}_q$$

$$x_A + x_B = x$$

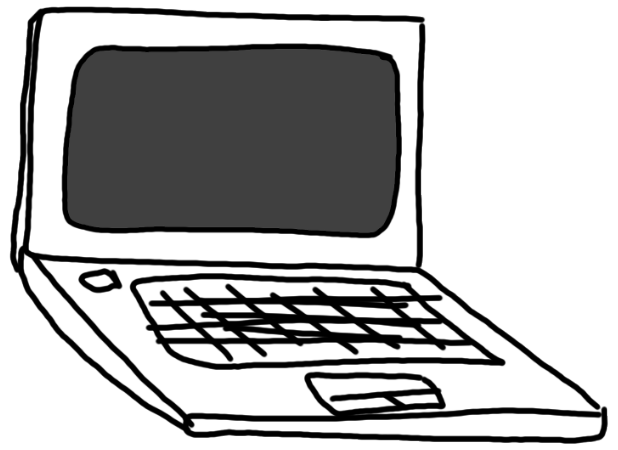


x_A $[x]$ x_B

Additive Secret Sharing

$$\boxed{?} \in \mathbb{Z}_q$$

$$x_A + \boxed{?} = \boxed{?}$$

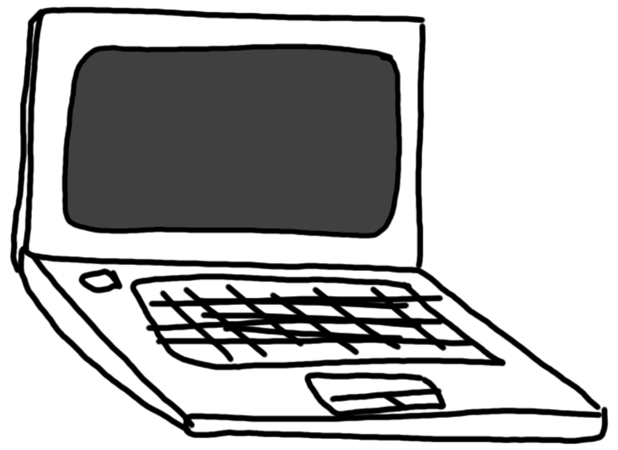


x_A $[\boxed{?}]$ $\boxed{?}$

Additive Secret Sharing

$$x \in \mathbb{Z}_q$$

$$x_A + x_B = x$$



x_A

$$[x]$$

x_B

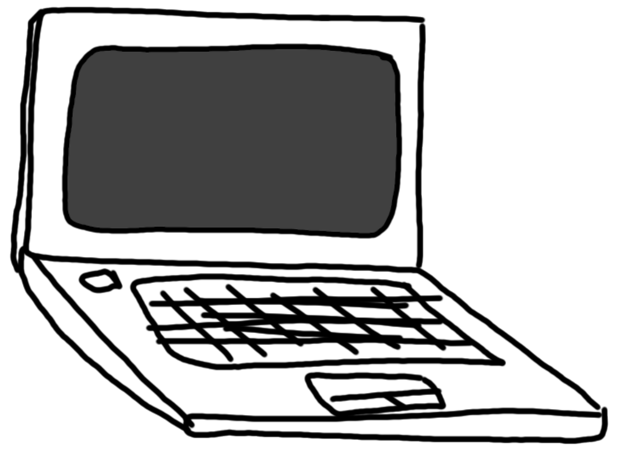
$$[y]$$

$$y_A + y_B = y$$

Additive Secret Sharing

$$x \in \mathbb{Z}_q$$

$$x_A + x_B = x$$



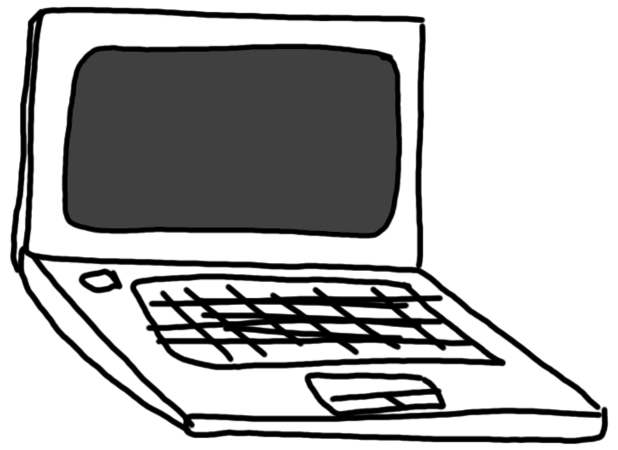
x_A $[x]$ x_B

y_A $[y]$ y_B

Additive Secret Sharing

$$x \in \mathbb{Z}_q$$

$$x_A + x_B = x$$



x_A $[x]$ x_B

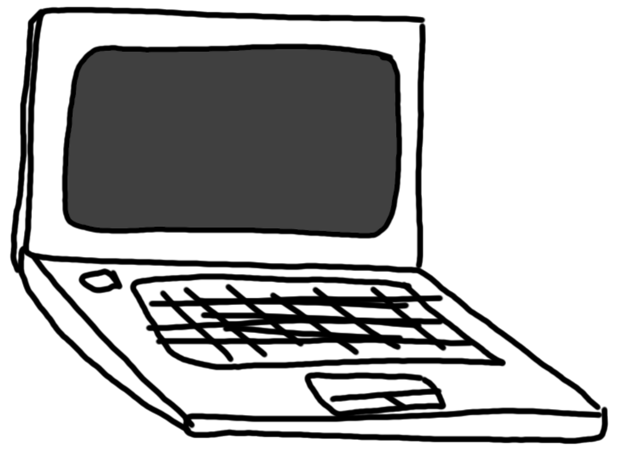
y_A $[y]$ y_B

$$[z = cx + y]$$

Additive Secret Sharing

$$x \in \mathbb{Z}_q$$

$$x_A + x_B = x$$



x_A	$[x]$	x_B
y_A	$[y]$	y_B
$z_A = cx_A + y_A$	$[z = cx + y]$	$z_B = cx_B + y_B$

Schnorr Key Generation

$$\text{SchnorrKeyGen}(\mathbb{G}, G, q):$$

$$\mathsf{sk} \leftarrow \mathbb{Z}_q$$

$$\mathsf{PK} = \mathsf{sk} \cdot G$$

$$\text{output } (\mathsf{sk}, \mathsf{PK})$$

<u>s</u>ecret <u>k</u>ey: kept private

<u>P</u>ublic <u>K</u>ey: exposed to the outside world

Distributing Schnorr KeyGen

$$\text{SchnorrKeyGen}(\mathbb{G}, G, q):$$
$$\mathsf{sk} \leftarrow \mathbb{Z}_q$$
$$\mathsf{PK} = \mathsf{sk} \cdot G$$
$$\text{output } (\mathsf{sk}, \mathsf{PK})$$

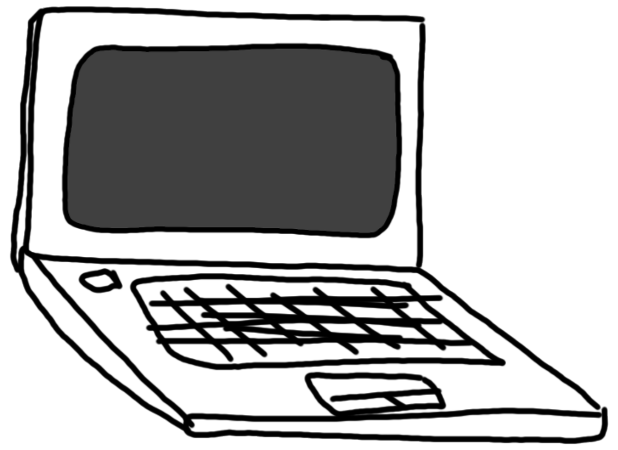
Distributing Schnorr KeyGen

$\mathrm{SchnorrKeyGen}(\mathbb{G}, G, q):$

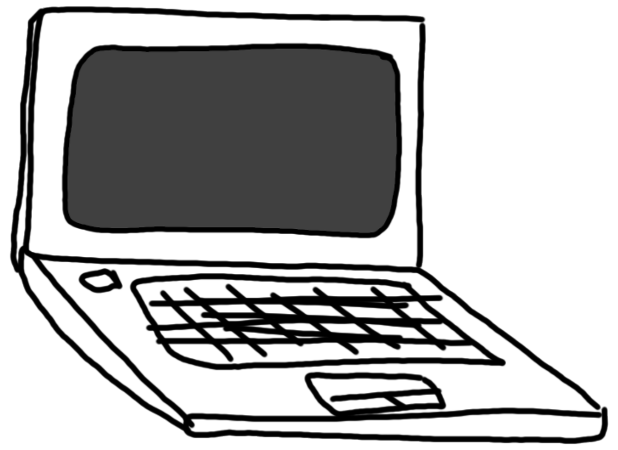
$\mathsf{sk} \leftarrow \mathbb{Z}_q$

$\mathsf{PK} = \mathsf{sk} \cdot G$

output $(\mathsf{sk}, \mathsf{PK})$



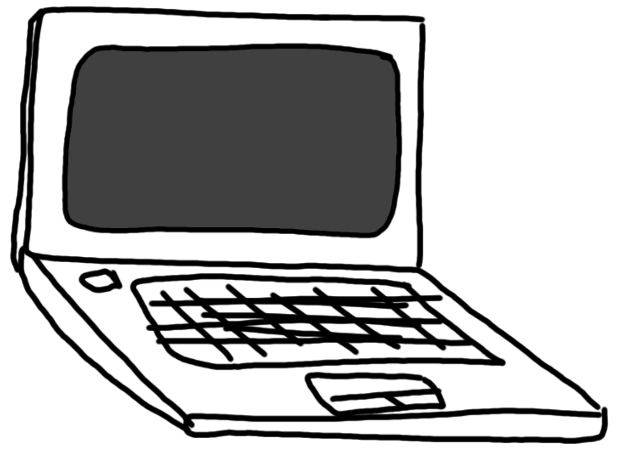
Distributing Schnorr KeyGen



$\text{SchnorrKeyGen}(\mathbb{G}, G, q):$

$\mathsf{sk} \leftarrow \mathbb{Z}_q$	$\mathsf{sk}_\mathsf{A} \leftarrow \mathbb{Z}_q$	$\mathsf{sk}_\mathsf{B} \leftarrow \mathbb{Z}_q$
$\mathsf{PK} = \mathsf{sk} \cdot G$		
$\text{output } (\mathsf{sk}, \mathsf{PK})$		

Distributing Schnorr KeyGen

		
$\mathrm{SchnorrKeyGen}(\mathbb{G}, G, q):$		
$\mathsf{sk} \leftarrow \mathbb{Z}_q$	$\mathsf{sk}_A \leftarrow \mathbb{Z}_q$	$\mathsf{sk}_B \leftarrow \mathbb{Z}_q$
$\mathsf{PK} = \mathsf{sk} \cdot G$	$\mathsf{PK}_A = \mathsf{sk}_A \cdot G$	$\mathsf{PK}_B = \mathsf{sk}_B \cdot G$
output $(\mathsf{sk}, \mathsf{PK})$		

Distributing Schnorr KeyGen

$\text{SchnorrKeyGen}(\mathbb{G}, G, q):$

$\mathsf{sk} \leftarrow \mathbb{Z}_q$

$\mathsf{PK} = \mathsf{sk} \cdot G$

output $(\mathsf{sk}, \mathsf{PK})$

$\mathsf{sk}_\mathsf{A} \leftarrow \mathbb{Z}_q$

$\mathsf{PK}_\mathsf{A} = \mathsf{sk}_\mathsf{A} \cdot G$

output $(\mathsf{sk}_\mathsf{A}, \mathsf{PK}_\mathsf{A})$

$\mathsf{sk}_\mathsf{B} \leftarrow \mathbb{Z}_q$

$\mathsf{PK}_\mathsf{B} = \mathsf{sk}_\mathsf{B} \cdot G$

output $(\mathsf{sk}_\mathsf{B}, \mathsf{PK}_\mathsf{B})$

Distributing Schnorr KeyGen

$\mathrm{SchnorrKeyGen}(\mathbb{G}, G, q):$

$\mathsf{sk} \leftarrow \mathbb{Z}_q$

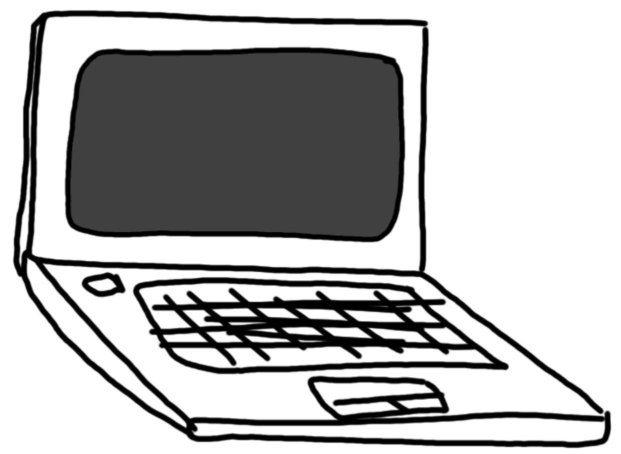
$\mathsf{PK} = \mathsf{sk} \cdot G$

output $(\mathsf{sk}, \mathsf{PK})$

$\mathsf{sk}_\mathsf{A} \leftarrow \mathbb{Z}_q$

$\mathsf{PK}_\mathsf{A} = \mathsf{sk}_\mathsf{A} \cdot G$

output $(\mathsf{sk}_\mathsf{A}, \mathsf{PK}_\mathsf{A})$



$\mathsf{sk}_\mathsf{B} \leftarrow \mathbb{Z}_q$

$\mathsf{PK}_\mathsf{B} = \mathsf{sk}_\mathsf{B} \cdot G$

output $(\mathsf{sk}_\mathsf{B}, \mathsf{PK}_\mathsf{B})$

How are the values related?

$$\mathsf{sk} = \mathsf{sk}_\mathsf{A} + \mathsf{sk}_\mathsf{B}$$

$$\mathsf{PK} = \mathsf{PK}_\mathsf{A} + \mathsf{PK}_\mathsf{B}$$

Distributing Schnorr KeyGen

$\text{SchnorrKeyGen}(\mathbb{G}, G, q):$

$\mathsf{sk} \leftarrow \mathbb{Z}_q$

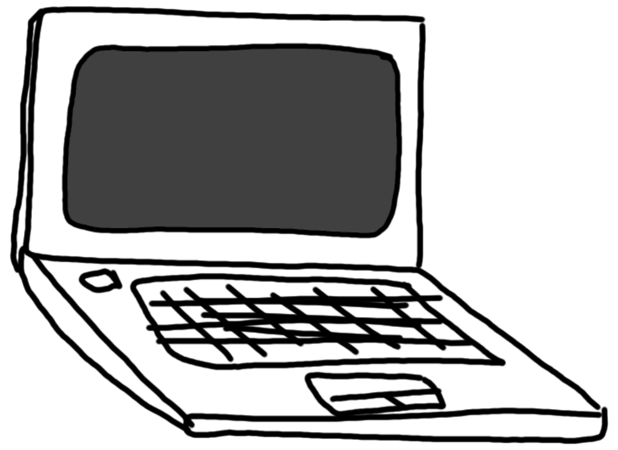
$\mathsf{PK} = \mathsf{sk} \cdot G$

$\text{output } (\mathsf{sk}, \mathsf{PK})$

$\mathsf{sk_A} \leftarrow \mathbb{Z}_q$

$\mathsf{PK_A} = \mathsf{sk_A} \cdot G$

$\text{output } (\mathsf{sk_A}, \mathsf{PK_A})$



$\mathsf{sk_B} \leftarrow \mathbb{Z}_q$

$\mathsf{PK_B} = \mathsf{sk_B} \cdot G$

$\text{output } (\mathsf{sk_B}, \mathsf{PK_B})$

How are the values related?

$\mathsf{sk} = \mathsf{sk_A} + \mathsf{sk_B}$

$\mathsf{PK} = \mathsf{PK_A} + \mathsf{PK_B}$

Distributing Schnorr KeyGen

$\text{SchnorrKeyGen}(\mathbb{G}, G, q):$

$\mathsf{sk} \leftarrow \mathbb{Z}_q$

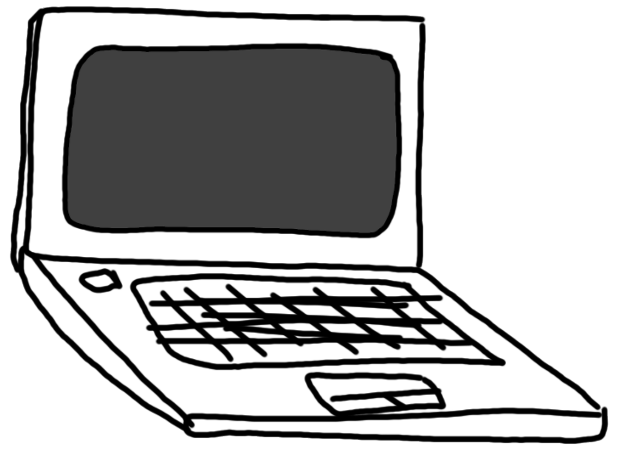
$\mathsf{PK} = \mathsf{sk} \cdot G$

output $(\mathsf{sk}, \mathsf{PK})$

$\mathsf{sk}_\mathsf{A} \leftarrow \mathbb{Z}_q$

$\mathsf{PK}_\mathsf{A} = \mathsf{sk}_\mathsf{A} \cdot G$

output $(\mathsf{sk}_\mathsf{A}, \mathsf{PK}_\mathsf{A})$



$\mathsf{sk}_\mathsf{B} \leftarrow \mathbb{Z}_q$

$\mathsf{PK}_\mathsf{B} = \mathsf{sk}_\mathsf{B} \cdot G$

output $(\mathsf{sk}_\mathsf{B}, \mathsf{PK}_\mathsf{B})$

How are the values related?

$\mathsf{sk} = \mathsf{sk}_\mathsf{A} + \mathsf{sk}_\mathsf{B}$

$\mathsf{PK} = \mathsf{PK}_\mathsf{A} + \mathsf{PK}_\mathsf{B}$

Distributing Schnorr KeyGen

$\mathrm{SchnorrKeyGen}(\mathbb{G}, G, q):$

$\boxed{?} \leftarrow \mathbb{Z}_q$

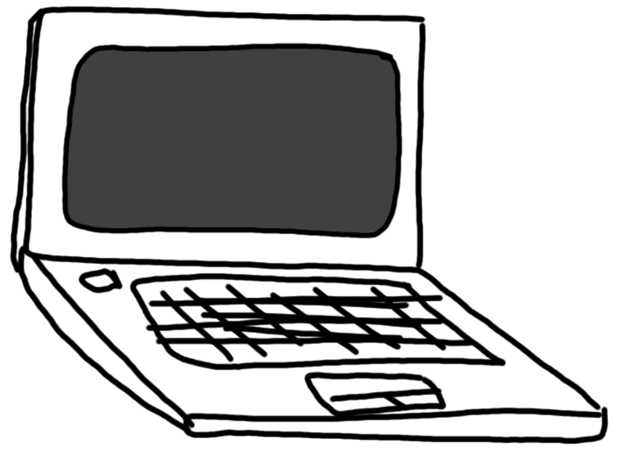
$\mathsf{PK} = \boxed{?} \cdot G$

output $(\boxed{?}, \mathsf{PK})$

$\mathsf{sk}_A \leftarrow \mathbb{Z}_q$

$\mathsf{PK}_A = \mathsf{sk}_A \cdot G$

output $(\mathsf{sk}_A, \mathsf{PK}_A)$



$\boxed{?} \leftarrow \mathbb{Z}_q$

$\mathsf{PK}_B = \boxed{?} \cdot G$

output $(\boxed{?}, \mathsf{PK}_B)$

How are the values related?

$\boxed{?} = \mathsf{sk}_A + \boxed{?}$

$\mathsf{PK} = \mathsf{PK}_A + \mathsf{PK}_B$

Distributing Schnorr KeyGen

$\mathrm{SchnorrKeyGen}(\mathbb{G}, G, q):$

$\boxed{?} \leftarrow \mathbb{Z}_q$

$\mathsf{PK} = \boxed{?} \cdot G$

output $(\boxed{?}, \mathsf{PK})$

$\mathsf{sk_A} \leftarrow \mathbb{Z}_q$

$\mathsf{PK_A} = \mathsf{sk_A} \cdot G$

output $(\mathsf{sk_A}, \mathsf{PK_A})$

$\boxed{?} \leftarrow \mathbb{Z}_q$

$\mathsf{PK_B} = \boxed{?} \cdot G$

output $(\boxed{?}, \mathsf{PK_B})$

How are the values related?

$\boxed{?} = \mathsf{sk_A} + \boxed{?}$

$\mathsf{PK} = \mathsf{PK_A} + \mathsf{PK_B}$

Note:

Computing $\mathsf{sk_B}$ given $\mathsf{PK_B}$ is just as hard as computing sk given PK

Distributing Schnorr KeyGen

$$\begin{aligned}
&\text{SchnorrKeyGen}(\mathbb{G}, G, q) : \\
&\quad \boxed{[\mathsf{sk}]} \leftarrow \mathbb{Z}_q \\
&\quad \mathsf{PK} = \boxed{[\mathsf{sk}]} \cdot G \\
&\quad \text{output } (\mathsf{sk}, \mathsf{PK})
\end{aligned}$$

Schnorr Signing

$\mathsf{SchnorrSign}(\mathsf{sk}, m):$

$$k \leftarrow \mathbb{Z}_q$$

$$R = k \cdot G$$

$$e = H(R \| m)$$

$$s = k - \mathsf{sk} \cdot e$$

$$\sigma = (s, R)$$

$$\text{output } \sigma$$

Schnorr Signing

$$\mathrm{SchnorrSign}(\mathsf{sk}, m):$$

$$k \leftarrow \mathbb{Z}_q$$

$$R = k \cdot G$$

Identical to KeyGen
Same protocol applies

$$e = H(R \| m)$$

$$s = k - \mathsf{sk} \cdot e$$

$$\sigma = (s, R)$$

$$\text{output } \sigma$$

Schnorr Signing

$$\mathrm{SchnorrSign}([\mathsf{sk}], m):$$

$$[k] \leftarrow \mathbb{Z}_q$$

$$R = [k] \cdot G$$

$$e = H(R \| m)$$

$$s = k - \mathsf{sk} \cdot e$$

$$\sigma = (s, R)$$

$$\text{output } \sigma$$

Identical to KeyGen
Same protocol applies

Schnorr Signing

$\mathsf{SchnorrSign}([\mathsf{sk}], m):$

$$\boxed{\begin{aligned}[k] &\leftarrow \mathbb{Z}_q \\ R &= [k] \cdot G\end{aligned}}$$

Identical to KeyGen
Same protocol applies

$$e = H(R \| m)$$

$$\boxed{[s] = [k] - [\mathsf{sk}] \cdot e}$$

Linear function of k, sk
Make use of linearity of secret sharing scheme

$$\sigma = (s, R)$$

output σ

3 Round Schnorr Signing

Folklore, [Lindell 22]

Input : $\mathsf{pk} = [\mathsf{sk}] \cdot G$, $[\mathsf{sk}]$, $[k]$

Establish $R = [k] \cdot G$

Round 1	Exchange Commit $\left(R_i = [k]_i \cdot G\right)$
Round 2	Release R_i, set $R = \Sigma_i\ R_i$
Round 3	Reveal $s = [\mathsf{sk}] \cdot H(m, R) + [k]$

Output (R, s)

(Threshold) Schnorr in Practice?

- Schnorr signatures are **old** (well-studied), **compact**, **fast** to generate and verify, and **easy to distribute with MPC (i.e. thresholdize)**
- However it was patented - major barrier for internet adoption
- Patent expired recently but the damage is done; adoption is increasing but much of the **internet infrastructure does not support Schnorr**
- Instead, **ECDSA** is widely deployed in its place—similar performance and security, and patent-free but MPC-unfriendly

ECDSA

- **E**lliptic **C**urve **D**igital **S**ignature **A**lgorithm
- Devised by Scott Vanstone in 1992, standardised by NIST
- Differs from Schnorr enough so that patent doesn't apply
- Widespread adoption across the internet

Threshold ECDSA: Challenges

$\mathrm{SchnorrSign}(\mathsf{sk}, m):$

$$k \leftarrow \mathbb{Z}_q$$
$$R = k \cdot G$$
$$e = H(R \| m)$$
$$s = k - \mathsf{sk} \cdot e$$
$$\sigma = (s, R)$$

output σ

$\mathrm{ECDSASign}(\mathsf{sk}, m):$

$$k \leftarrow \mathbb{Z}_q$$
$$R = k \cdot G$$
$$e = H(m)$$

Threshold ECDSA: Challenges

$\mathsf{SchnorrSign}(\mathsf{sk}, m):$

$$k \leftarrow \mathbb{Z}_q$$

$$R = k \cdot G$$

$$e = H(R \| m)$$

$$s = k - \mathsf{sk} \cdot e$$

$$\sigma = (s, R)$$

output σ

$\mathsf{ECDSASign}(\mathsf{sk}, m):$

$$k \leftarrow \mathbb{Z}_q$$

$$R = k \cdot G$$

$$e = H(m)$$

$$s = \frac{e + \mathsf{sk} \cdot r_x}{k}$$

output $\sigma = (s, R)$

Threshold ECDSA: Challenges

$$\mathrm{ECDSASign}(\mathsf{sk}, m):$$

$$k \leftarrow \mathbb{Z}_q$$

$$R = k \cdot G$$

$$e = H(m)$$

$$\boxed{s = \frac{e + \mathsf{sk} \cdot r_x}{k}}$$

$$\text{output } \sigma = (s, R)$$

Threshold ECDSA: Challenges

$$\mathrm{ECDSASign}(\mathsf{sk}, m):$$

$$k \leftarrow \mathbb{Z}_q$$

$$R = k \cdot G$$

$$e = H(m)$$

$$s = \frac{e + \boxed{\mathsf{sk}} \cdot r_x}{\boxed{k}}$$

$$\text{output } \sigma = (s, R)$$

Multiplication of secret values

Threshold ECDSA: Challenges

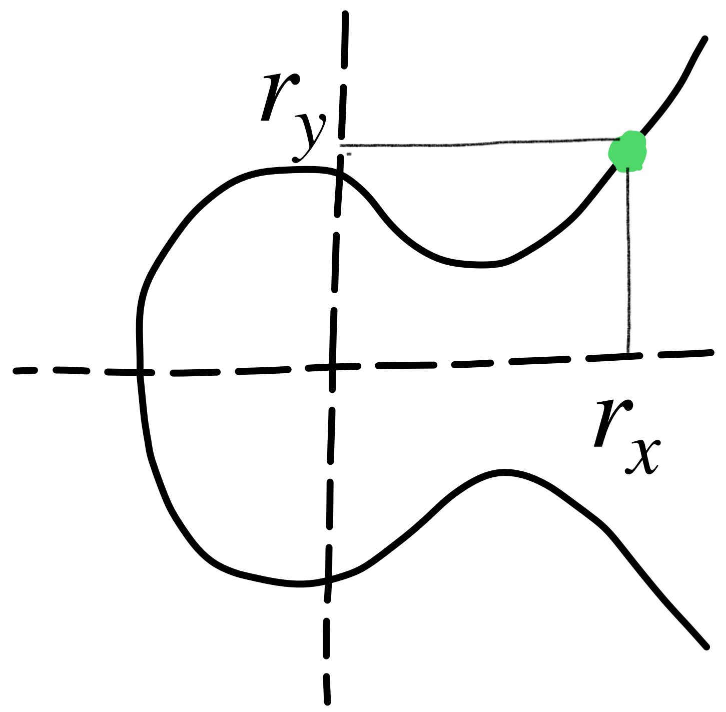
$$
\begin{aligned}
&\mathrm{ECDSASign}(\mathsf{sk}, m):\\
&k \leftarrow \mathbb{Z}_q\\
&R = k \cdot G\\
&e = H(m)\\
&s = \frac{e + \mathsf{sk} \cdot r_x}{\boxed{k}}\\
&\text{output } \sigma = (s, R)
\end{aligned}
$$

Multiplication of secret values

Division (Modular inverse)

Threshold ECDSA: Challenges

$R = (r_x, r_y)$



$$\mathsf{ECDSASign}(\mathsf{sk}, m):$$

$$k \leftarrow \mathbb{Z}_q$$

$$R = k \cdot G$$

$$e = H(m)$$

$$s = \frac{e + \mathsf{sk} \cdot r_x}{\boxed{k}}$$

$$\text{output } \sigma = (s, R)$$

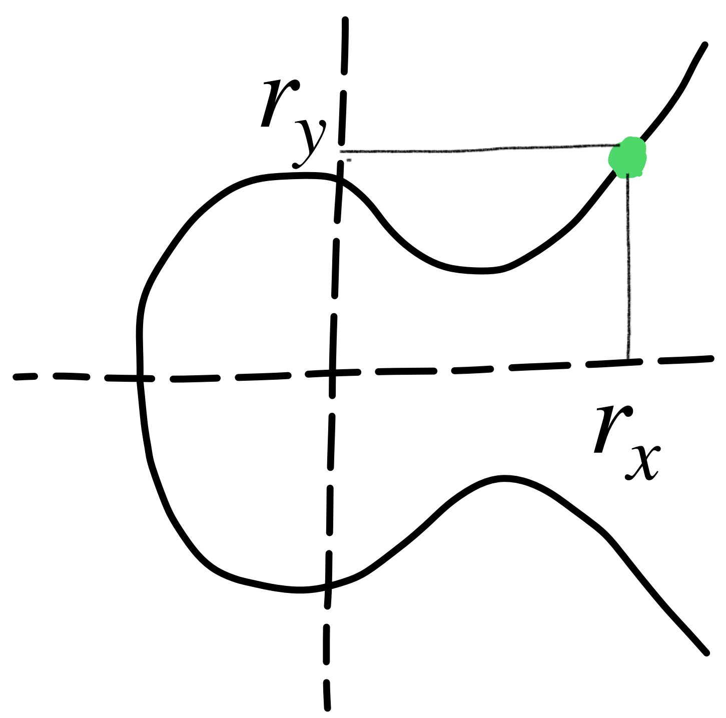
Multiplication of secret values

x-coordinate of ***R*** (not secret)

Division (Modular inverse)

Threshold ECDSA: Challenges

$R = (r_x, r_y)$



$$\mathsf{ECDSASign}(\mathsf{sk}, m):$$
$$k \leftarrow \mathbb{Z}_q$$
$$R = k \cdot G$$
$$e = H(m)$$
$$s = \frac{e + \mathsf{sk} \cdot r_x}{\boxed{k}}$$
$$\text{output } \sigma = (s, R)$$

There is no one-size-fits-all solution

Multiplication of secret values

x-coordinate of $\boldsymbol{R}$ (not secret)

Division (Modular inverse)

Threshold ECDSA: State of the Art

- Rough costs with 256-bit curve, for each additional party (computation aggregated across [Gavenda 21, XAXYC 21, BMP 22]):

Protocol	Tool	Rounds	Bandwidth (KB)	Computation (ms)
[DKLs 19]	OT	$\log(t) + 6$	90	<10
[HLNR 18/23]	OT+	7	40	50—100
[CGGMP 20]	Paillier	4	15	Hundreds
[GG 18]		8	7	Hundreds
[CCLST20, YCX21]	Class Groups	4	4	> 1000

Threshold ECDSA: Goal

- Rough costs with 256-bit curve, for each additional party (computation aggregated across [Gavenda 21, XAXYC 21, BMP 22]):

Protocol	Tool	Bandwidth (KB)	Computation (ms)
[DKLs 19]	OT	90	<10
[HLNR 18/23]	OT+	40	50—100
[CGGMP 20]	Paillier	15	Hundreds
[GG 18]		7	Hundreds
[CCLST20, YCX21]	Class Groups	4	> 1000

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<u>This work:</u>
3 Round Signing
from
2 round MULT

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<u>This work:</u>
3 Round Signing
from
2 round MULT

mild/no overhead

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<u>This work:</u>
3 Round Signing from 2 round MULT

mild/no overhead

<u>Insight</u>: well-chosen correlation+simple consistency check

Threshold ECDSA: Goal

- Rough costs with 256-bit curve, for each additional party (computation aggregated across [Gavenda 21, XAXYC 21, BMP 22]):

Protocol	Tool	Bandwidth (KB)	Computation (ms)
[DKLs 23]	OT	60	<10
[HLNR 18/23]	OT+	40	50—100
[CGGMP 20]	Paillier	15	Hundreds
[GG 18]	Paillier	7	Hundreds
[CCLST20, YCX21]	Class Groups	4	> 1000

This work:
3 Round Signing from 2 round MULT

mild/no overhead

Insight:
well-chosen correlation+simple consistency check

Intro		How to distribute ECDSA			Tradeoffs
MPC for Schnorr is easy	but not ECDSA	Rewriting ECDSA	ECDSA Tuples	2P-MUL + Consistency	

MPC for ECDSA

- In principle: can use generic MPC to compute $[s] = (e + [\mathsf{sk}] \cdot r_x) \cdot [k^{-1}]$
- However, computing $[k^{-1}]$ given $[k]$ naively is prohibitively expensive
- Rewrite ECDSA signing equation to an “MPC-friendly” equivalent
  i.e. only additions and multiplications of secret values
- Bar-Ilan Beaver 89: Inversion outside MPC

Rewriting ECDSA for MPC

[Lindell Nof Ranellucci 18] [Abram Nof Orlandi Scholl Shlomovits 22]

$$\text{ECDSASign}(\mathsf{sk}, m):$$

$$[k] \leftarrow \mathbb{Z}_q$$

$$R = [k] \cdot G$$

$$e = H(m)$$

$$s = \frac{e + [\mathsf{sk}] \cdot r_x}{[k]}$$

$$\text{output } \sigma = (s, R)$$

Rewriting ECDSA for MPC

[Lindell Nof Ranellucci 18] [Abram Nof Orlandi Scholl Shlomovits 22]

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$$s = \frac{e + [\mathsf{sk}] \cdot r_x}{[k]} \cdot \frac{[\phi]}{[\phi]}$$

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Rewriting ECDSA for MPC

[Lindell Nof Ranellucci 18] [Abram Nof Orlandi Scholl Shlomovits 22]

$$\mathrm{ECDSASign}(\mathsf{sk}, m):$$

$$[k] \leftarrow \mathbb{Z}_q$$

$$R = [k] \cdot G$$

$$e = H(m)$$

$$\alpha = (e + [\mathsf{sk}] \cdot r_x)\,[\phi]$$

$$\beta = [k][\phi]$$

$$s =$$

$$\text{output } \sigma = (s, R)$$

Rewriting ECDSA for MPC

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$$\alpha = (e + [\mathsf{sk}] \cdot r_x)\,[\phi]$$

$$\beta = [k][\phi]$$

$$s = \frac{\alpha}{\beta}$$

output $\sigma = (s, R)$

Rewriting ECDSA for MPC

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Rewriting ECDSA for MPC

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$$[k] \leftarrow \mathbb{Z}_q$$

$$R = [k] \cdot G$$

$$e = H(m)$$

$$[\phi] \leftarrow \mathbb{Z}_q$$

Public values

$$\alpha = (e + [\mathsf{sk}] \cdot r_x)\,[\phi]$$

$$\beta = [k][\phi]$$

$$s = \frac{\alpha}{\beta}$$

$$\text{output } \sigma = (s, R)$$

Rewriting ECDSA for MPC

[Lindell Nof Ranellucci 18] [Abram Nof Orlandi Scholl Shlomovits 22]

$\mathrm{ECDSASign}(\mathsf{sk}, m):$

$$[k] \leftarrow \mathbb{Z}_q$$

$$R = [k] \cdot G$$

$$e = H(m)$$

$$[\phi] \leftarrow \mathbb{Z}_q$$

$$\alpha = (e + [\mathsf{sk}] \cdot r_x)\,[\phi]$$

$$\beta = [k][\phi]$$

Public values

Safe to reveal
β: because ϕ is OTP
α: because fixed by β, s

$$s = \frac{\alpha}{\beta}$$

output $\sigma = (s, R)$

Rewriting ECDSA for MPC

[Lindell Nof Ranellucci 18] [Abram Nof Orlandi Scholl Shlomovits 22]

$$\mathrm{ECDSASign}(\mathsf{sk}, m):$$

$$[k] \leftarrow \mathbb{Z}_q$$

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$$e = H(m)$$

$$[\phi] \leftarrow \mathbb{Z}_q$$

$$\alpha = (e + [\mathsf{sk}] \cdot r_x)\,[\phi]$$

$$\beta = [k][\phi]$$

$$s = \frac{\alpha}{\beta}$$

$$\text{output } \sigma = (s, R)$$

Public values

Safe to reveal
β: because ϕ is OTP
α: because fixed by β, s

Secure mult: Only (nonlinear) combination of secret values

Signing from ECDSA Tuples

[Abram Nof Orlandi Scholl Shlomovits 22]

$[\mathsf{sk}]$ $[k]$ $[\phi]$ $[\phi k]$ $[\phi\mathsf{sk}]$

Signing from ECDSA Tuples

[Abram Nof Orlandi Scholl Shlomovits 22]

$[\mathsf{sk}]$ $[k]$ $[\phi]$ $[\phi k]$ $[\phi \mathsf{sk}]$

Round 1

Round 2

Establish $R = [k] \cdot G$

Signing from ECDSA Tuples

[Abram Nof Orlandi Scholl Shlomovits 22]

$[\mathsf{sk}]$ $[k]$ $[\phi]$ $[\phi k]$ $[\phi \mathsf{sk}]$

Round 1 Round 2	Establish $R = [k] \cdot G$
Round 3	Reveal $\alpha = e + r_x[\phi \mathsf{sk}]$ and $\beta = [\phi k]$

Output $(R, s = \alpha/\beta)$

ECDSA Tuple Generation

$[\mathsf{sk}][k]$

$[\phi]$

$[\phi k]$

$[\phi \mathsf{sk}]$

ECDSA Tuple Generation

Input : $[\mathsf{sk}][k]$

Sample : $[\phi]$

$[\phi k]$

$[\phi \mathsf{sk}]$

ECDSA Tuple Generation

Local

Input : $[\mathsf{sk}][k]$

Sample : $[\phi]$

$[\phi k]$

$[\phi \mathsf{sk}]$

ECDSA Tuple Generation

Local

$$\text{Input}: [\mathsf{sk}][k]$$
$$\text{Sample}: [\phi]$$

$$[\phi k] = \mathsf{MULT}([\phi], [k])$$
$$[\phi \mathsf{sk}] = \mathsf{MULT}([\phi], [\mathsf{sk}])$$

ECDSA Tuple Generation

Local

$$\text{Input}: [\mathsf{sk}][k]$$
$$\text{Sample}: [\phi]$$

Consistency: straightforward

$$[\phi k] = \mathsf{MULT}([\phi],[k])$$
$$[\phi \mathsf{sk}] = \mathsf{MULT}([\phi],[\mathsf{sk}])$$

ECDSA Tuple Generation

Local

$$\text{Input}: [\mathsf{sk}][k]$$
$$\text{Sample}: [\phi]$$

Consistency: straightforward

$$[\phi k] = \mathsf{MULT}([\phi], [k])$$
$$[\phi \mathsf{sk}] = \mathsf{MULT}([\phi], [\mathsf{sk}])$$

Verify:
$$[k] \cdot G = R$$
$$[\mathsf{sk}] \cdot G = \mathsf{pk}$$

ECDSA Tuple Generation

Local

$$\text{Input}: [\mathsf{sk}][k]$$
$$\text{Sample}: [\phi]$$

Consistency: straightforward

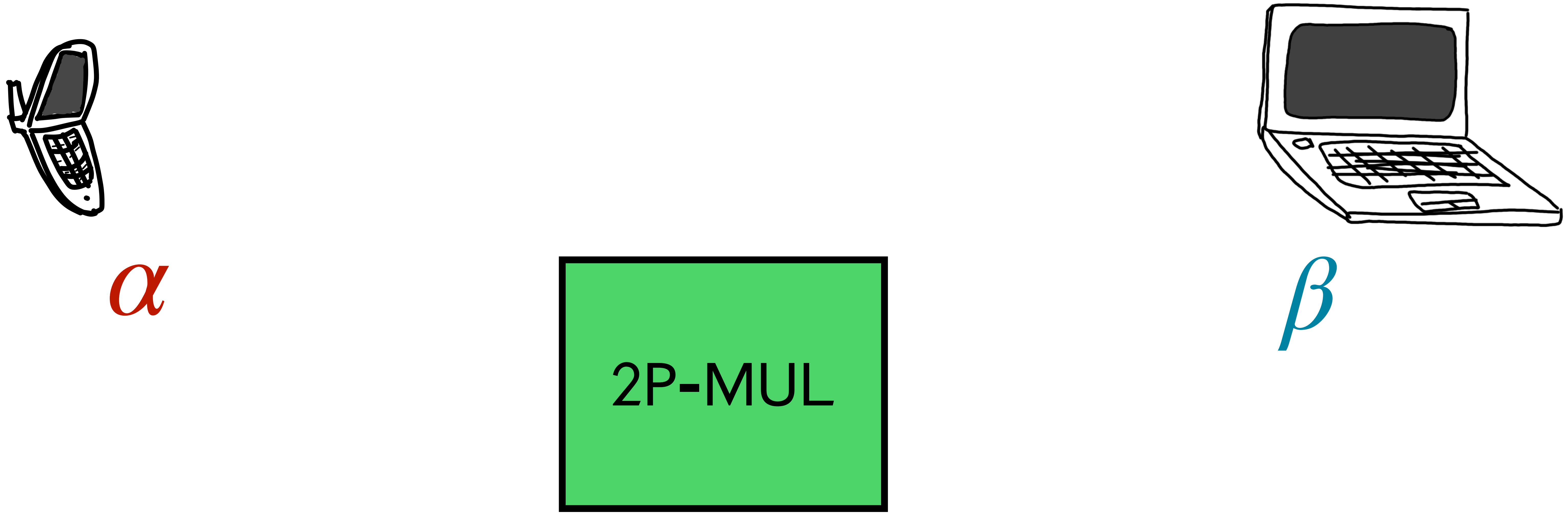
$$[\phi k] = \mathsf{MULT}([\phi], [k])$$
$$[\phi \mathsf{sk}] = \mathsf{MULT}([\phi], [\mathsf{sk}])$$

Previous works: ZK proofs, MACs, etc.
<u>This work</u>: Simple pairwise check

Verify:
$$[k] \cdot G = R$$
$$[\mathsf{sk}] \cdot G = \mathsf{pk}$$

Secure Two-Party Multiplication

a.k.a. OLE, Mult2Add



Secure Two-Party Multiplication

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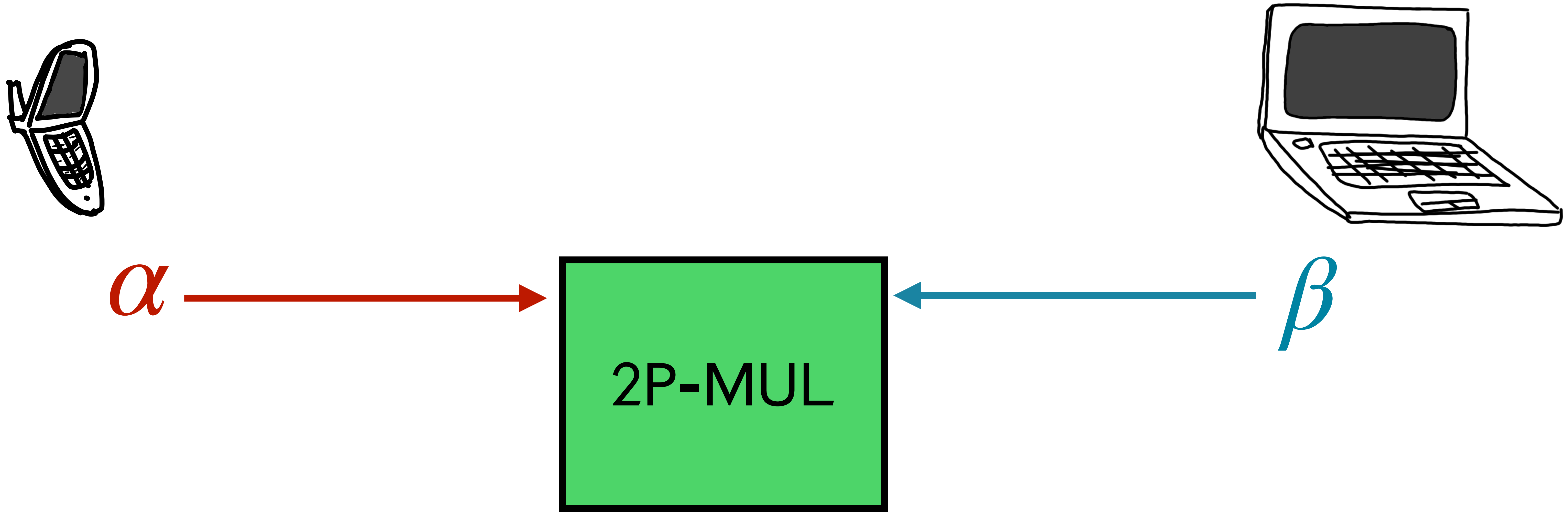

Secure Two-Party Multiplication

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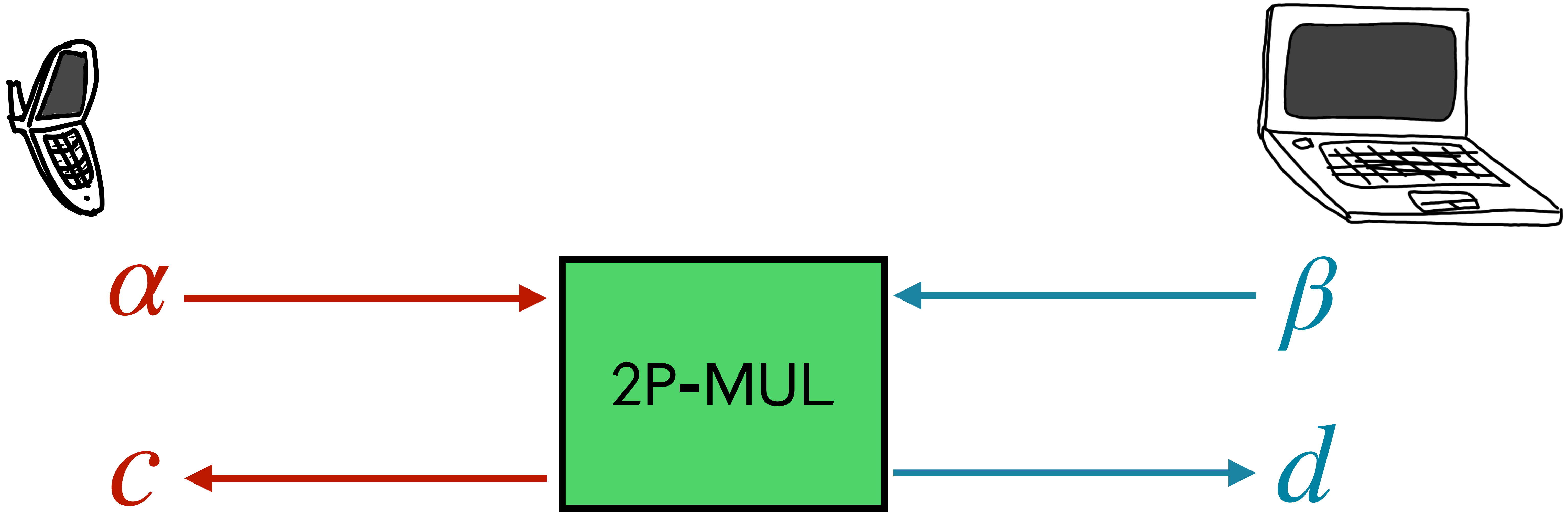

Secure Two-Party Multiplication

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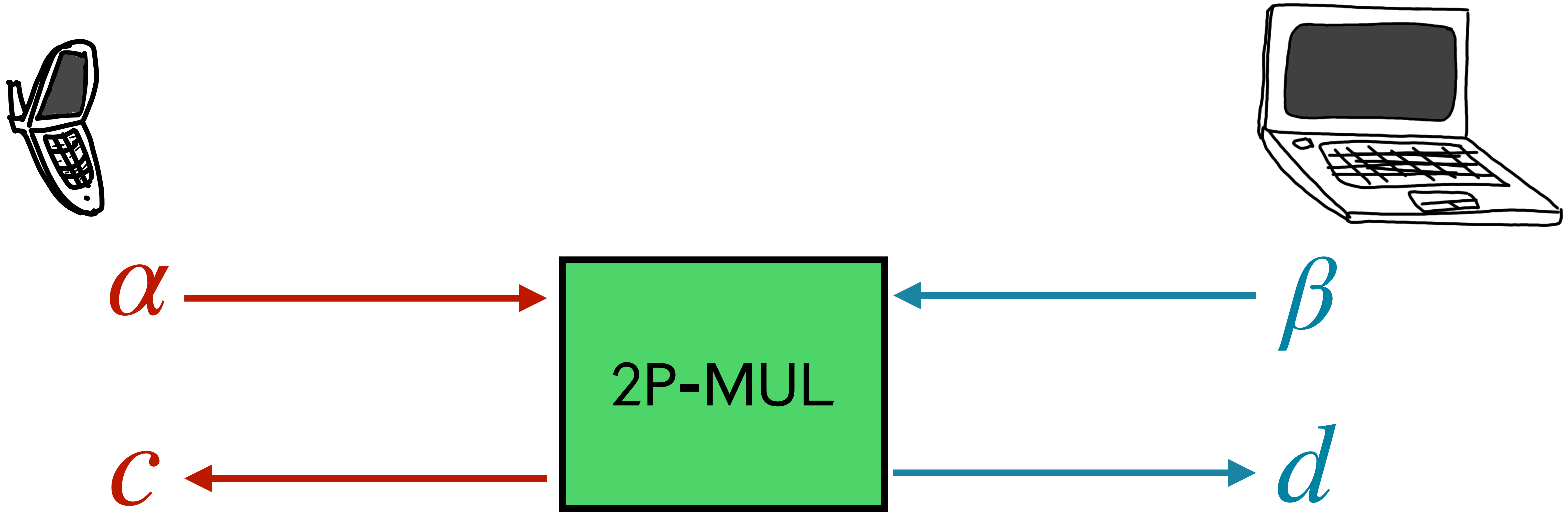

$$c + d = \alpha \cdot \beta$$

Secure Two-Party Multiplication

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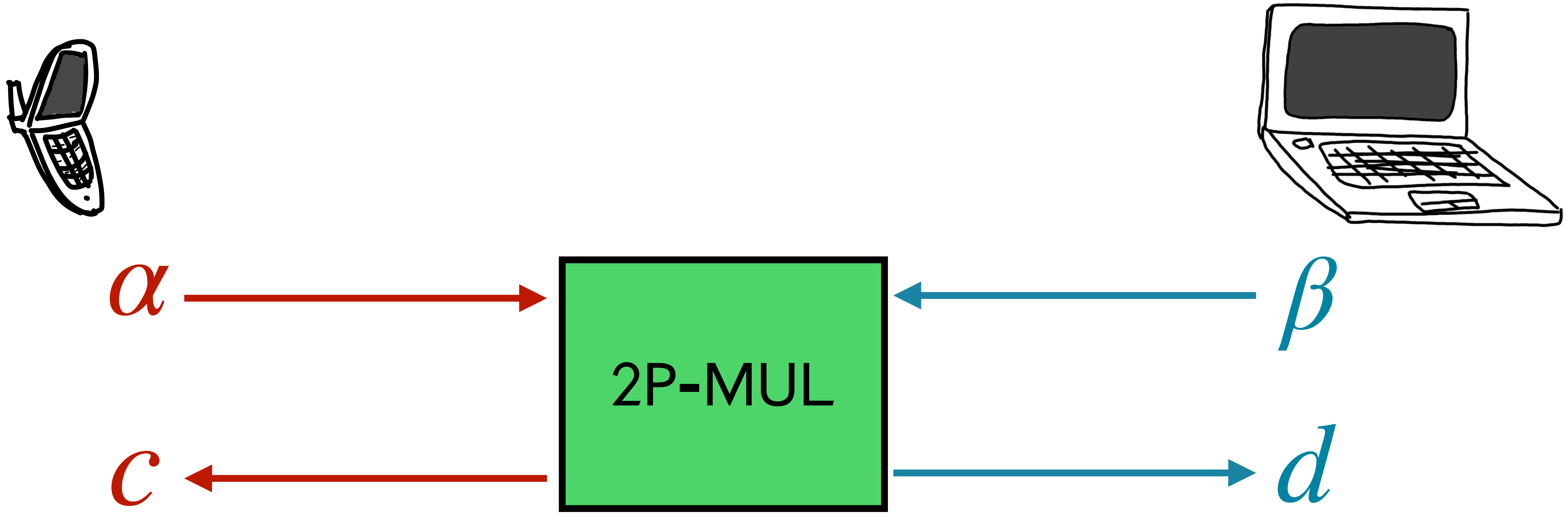
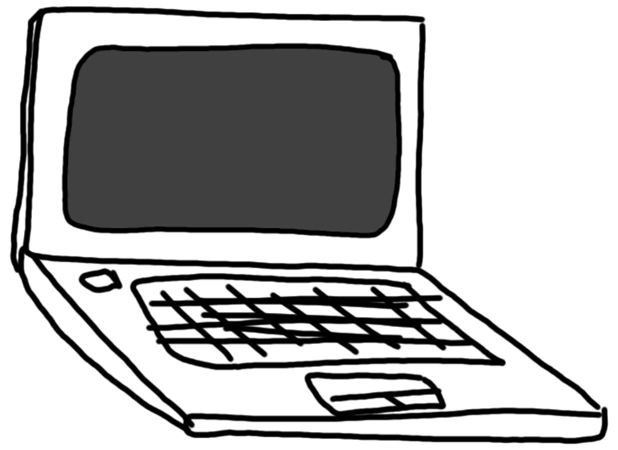


$$c + d = \alpha \cdot \beta$$

Gadget to split a product of secret inputs $\alpha\beta$ into additive secrets c, d

Instantiable efficiently from: OT, Paillier, Class Groups

Two-Round 2P-MUL



$$e = \mathsf{Enc}_1(\beta)$$

$\longleftarrow$

$\longrightarrow$

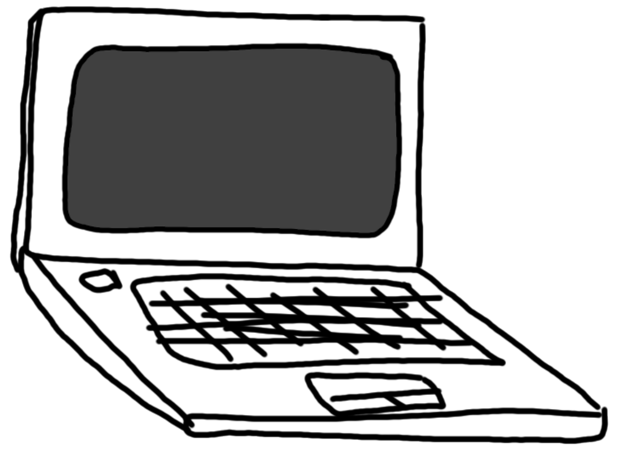
$$\mathsf{Enc}_2(e, \alpha)$$

c

d

$$c + d = \alpha \cdot \beta$$

Two-Round 2P-MUL



$e = \mathsf{Enc}_1(\beta)$

$\longleftarrow$

$\longrightarrow$

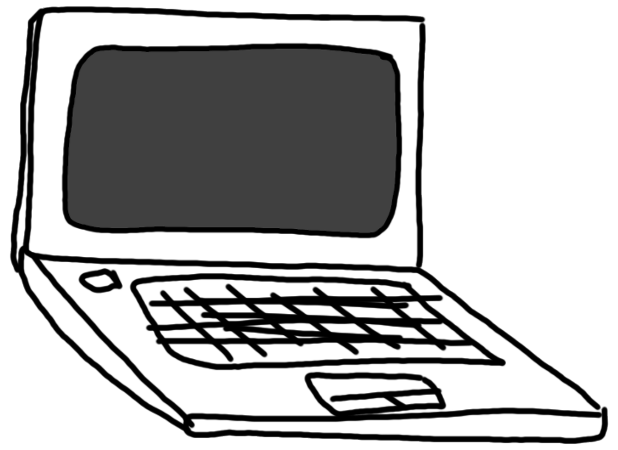
$\mathsf{Enc}_2(e, \alpha)$
$\mathsf{Enc}_2(e, \hat{\alpha})$

c

d

$$c + d = \alpha \cdot \beta$$
$$\hat{c} + \hat{d} = \hat{\alpha} \cdot \beta$$

Two-Round 2P-MUL



$$e = \mathsf{Enc}_1(\beta)$$

$$\mathsf{Enc}_2(e, \alpha)$$
$$\mathsf{Enc}_2(e, \hat{\alpha})$$

c

d

$$c + d = \alpha \cdot \beta$$
$$\hat{c} + \hat{d} = \hat{\alpha} \cdot \beta$$

Consistency
“for free”

ECDSA Tuple Generation

$$\text{Input} : [\mathsf{sk}][k]$$
$$\text{Sample} : [\phi]$$

Consistency: straightforward ✓

$$[\phi k] = \mathsf{MULT}([\phi], [k])$$
$$[\phi \mathsf{sk}] = \mathsf{MULT}([\phi], [\mathsf{sk}])$$

ECDSA Tuple Generation

$\text{Input} : [\mathsf{sk}][k]$
$\text{Sample} : [\phi]$

Consistency: straightforward ✓

$$[\phi k] = \mathsf{MULT}([\phi], [k])$$
$$[\phi \mathsf{sk}] = \mathsf{MULT}([\phi], [\mathsf{sk}])$$

Verify: $[k] \cdot G = R$
$[\mathsf{sk}] \cdot G = \mathsf{pk}$

Verifying Consistency w.r.t. $\mathbb{G}$

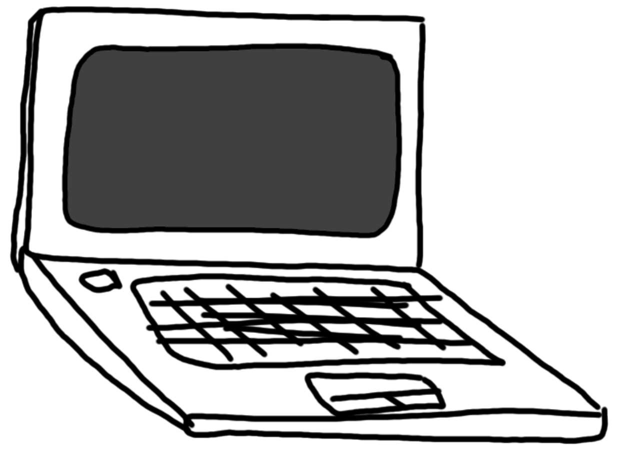
$\mathsf{MULT}([\phi], [k])$

Simplified :

ϕ

2P-MUL

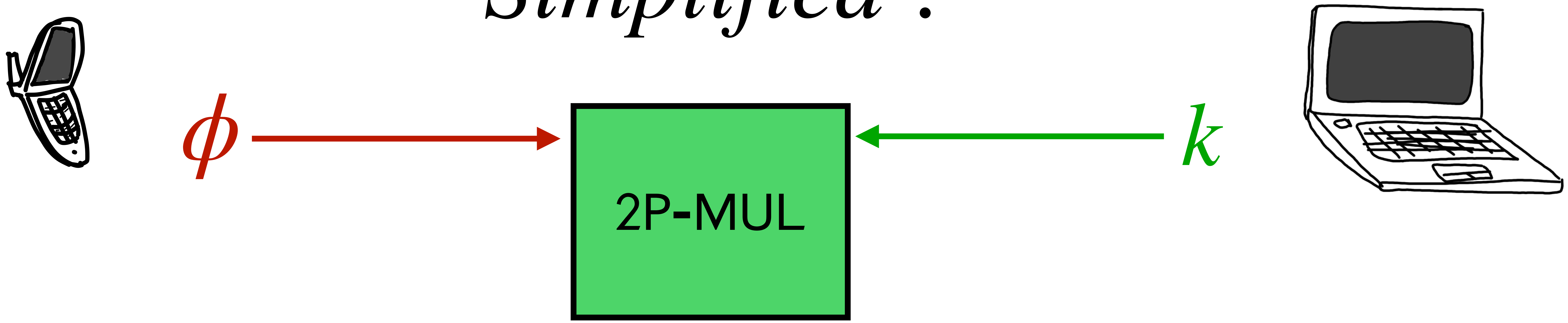
k



Verifying Consistency w.r.t. $\mathbb{G}$

$\mathsf{MULT}([\phi], [k])$

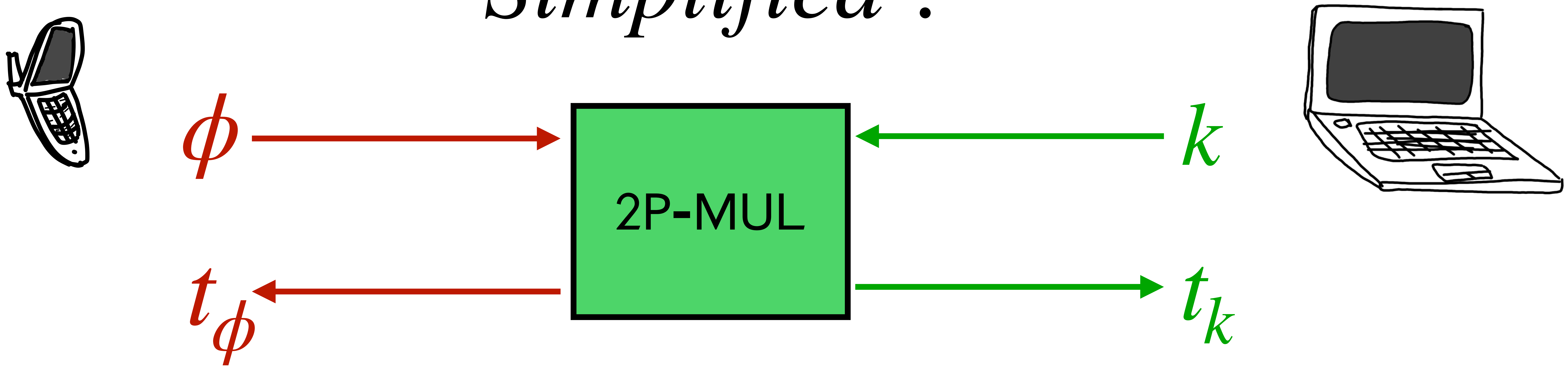
Simplified :



Verifying Consistency w.r.t. $\mathbb{G}$

$\mathsf{MULT}([\phi], [k])$

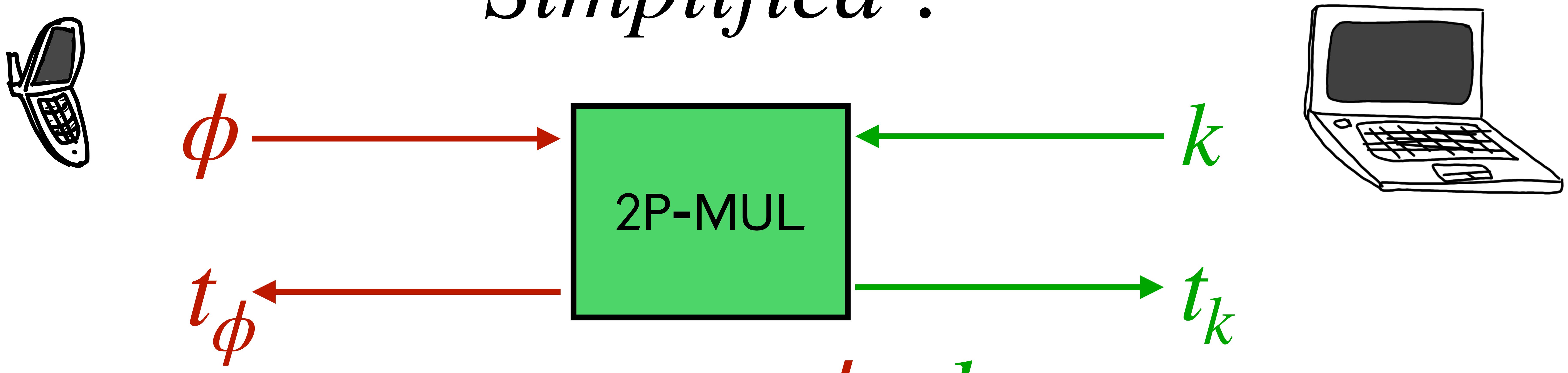
Simplified :



Verifying Consistency w.r.t. $\mathbb{G}$

$\mathsf{MULT}([\phi],[k])$

Simplified :

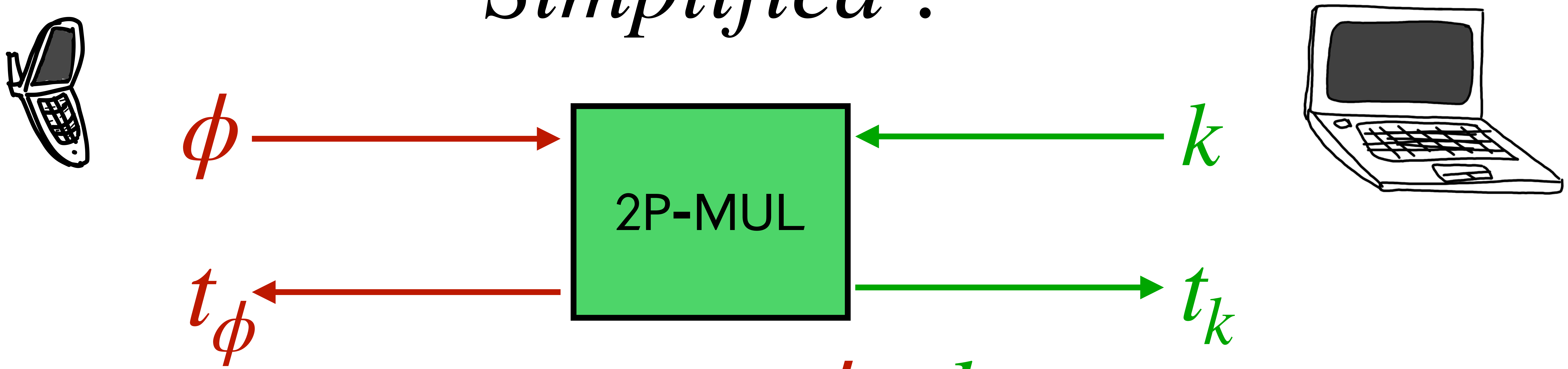


$$t_\phi + t_k = \phi \cdot k$$

Verifying Consistency w.r.t. $\mathbb{G}$

$\mathsf{MULT}([\phi], [k])$

Simplified :



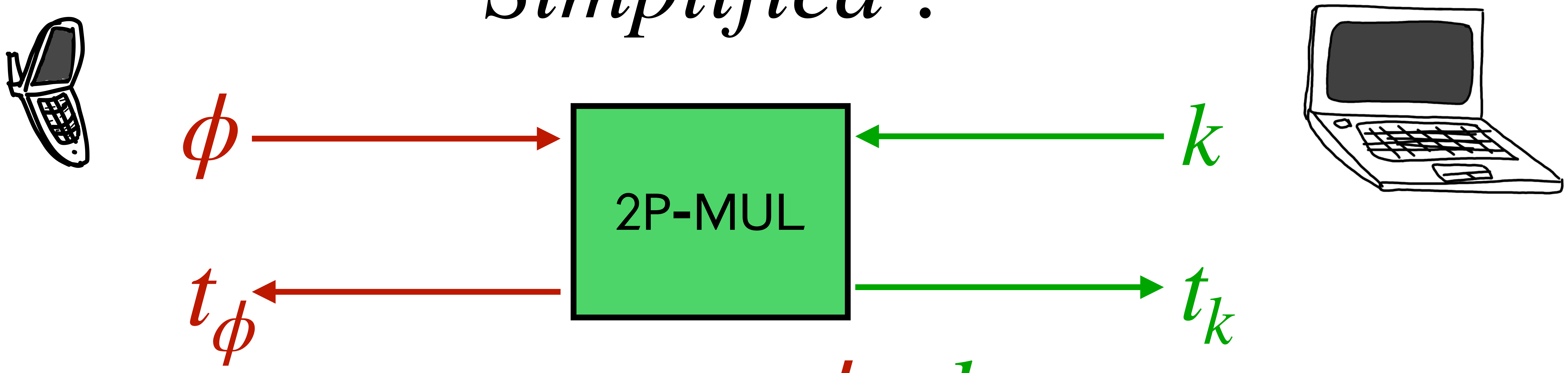
$$t_\phi + t_k = \phi \cdot k$$

Claim: $[k] \cdot G = R$

Verifying Consistency w.r.t. $\mathbb{G}$

$\mathsf{MULT}([\phi], [k])$

Simplified :



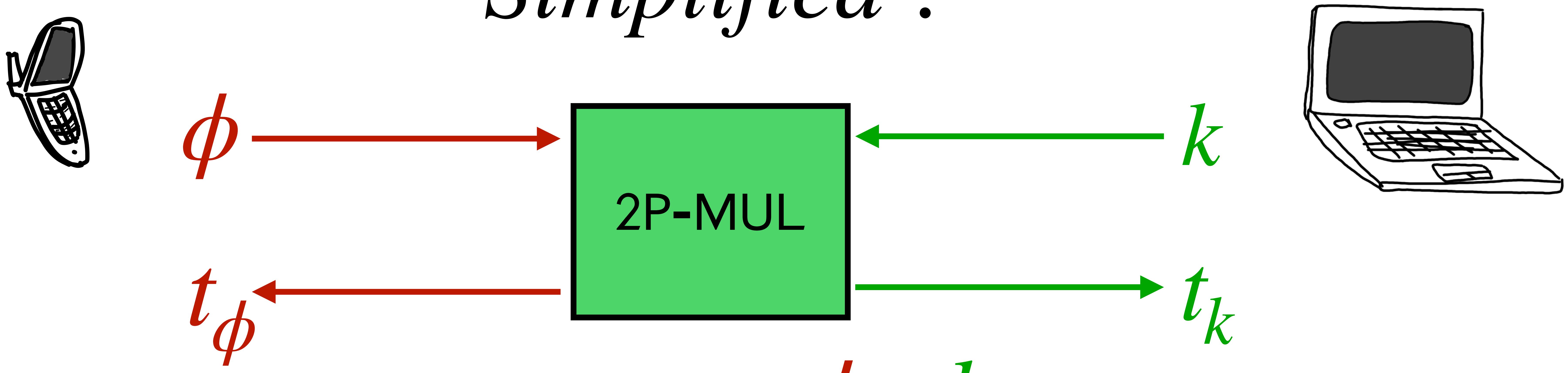
$$T_\phi = t_\phi \cdot G \qquad t_\phi + t_k = \phi \cdot k \qquad T_k = t_k \cdot G$$

Claim: $[k] \cdot G = R$

Verifying Consistency w.r.t. $\mathbb{G}$

$\text{MULT}([\phi], [k])$

Simplified :



$$t_\phi + t_k = \phi \cdot k$$

$$T_\phi = t_\phi \cdot G \qquad T_k = t_k \cdot G$$

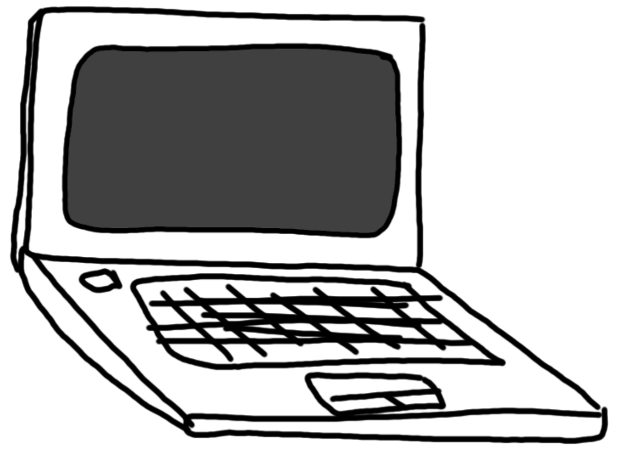
Claim: $[k] \cdot G = R$

$$T_k = \phi R - T_\phi$$

Verifying Consistency w.r.t. $\mathbb{G}$

MULT([ϕ], [k])

Simplified :



ϕ → 2P-MUL ← k

t_ϕ ← 2P-MUL → t_k

$$T_\phi = t_\phi \cdot G \qquad t_\phi + t_k = \phi \cdot k \qquad T_k = t_k \cdot G$$

Claim: $[k] \cdot G = R$

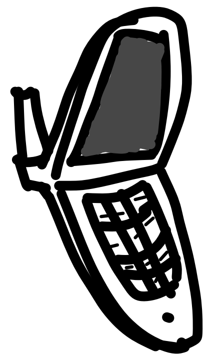
$$T_k = \phi R - T_\phi$$

T_k ←

Verifying Consistency w.r.t. $\mathbb{G}$

MULT([ϕ], [k])

Simplified :



$\phi \rightarrow$ 2P-MUL $\leftarrow k$

$t_\phi \leftarrow$ 2P-MUL $\rightarrow t_k$

$$T_\phi = t_\phi \cdot G \qquad t_\phi + t_k = \phi \cdot k \qquad T_k = t_k \cdot G$$

Claim: $[k] \cdot G = R$

$$\boxed{T_k} = \phi R - T_\phi$$

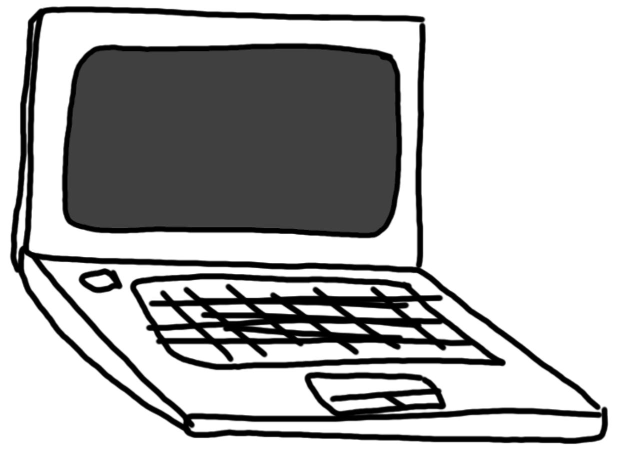
Match?

$\boxed{T_k}$

Verifying Consistency w.r.t. $\mathbb{G}$

$\mathsf{MULT}([\phi], [k])$

Simplified :



$\phi \longrightarrow$ 2P-MUL $\longleftarrow k$

$t_\phi \longleftarrow$ 2P-MUL $\longrightarrow t_k$

No information about k

$T_\phi = t_\phi \cdot G$

$t_\phi + t_k = \phi \cdot k$

$T_k = t_k \cdot G$

Claim: $[k] \cdot G = R$

$\boxed{T_k} = \phi R - T_\phi$

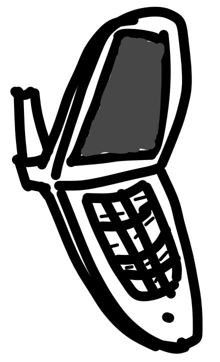
Match?

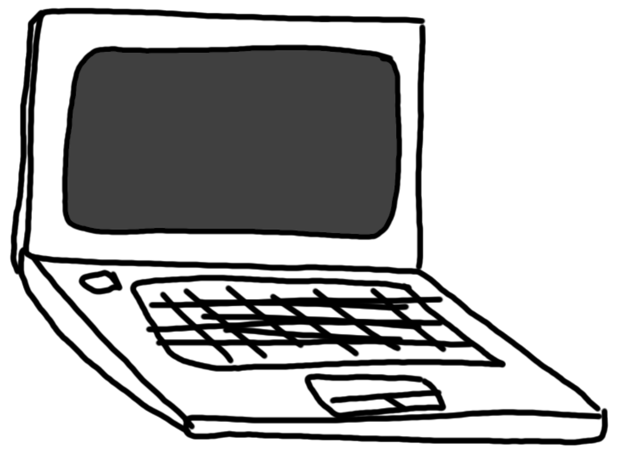
$\boxed{T_k}$

Verifying Consistency w.r.t. $\mathbb{G}$

$\mathsf{MULT}([\phi], [k])$

Simplified :





No information about k

$\phi \longrightarrow$ 2P-MUL $\longleftarrow k$

$t_\phi \longleftarrow$ 2P-MUL $\longrightarrow t_k$

$$T_\phi = t_\phi \cdot G \qquad t_\phi + t_k = \phi \cdot k \qquad T_k = t_k \cdot G$$

Claim: $[k] \cdot G = R$

$\boxed{T_k} = \phi R - T_\phi$

Match?

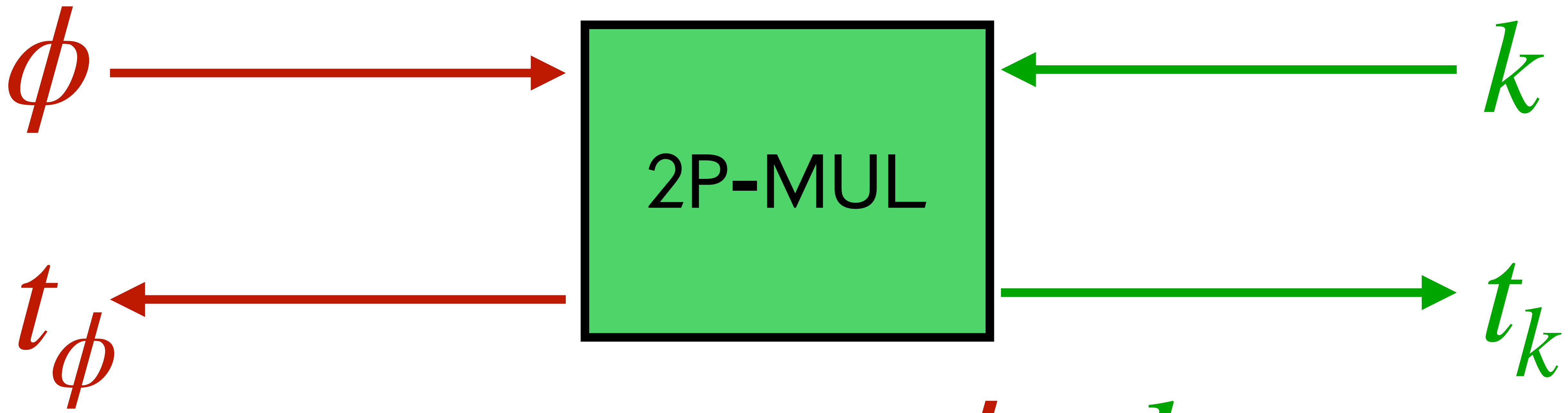
$\boxed{T_k}$

In case of cheat:
have to guess ϕ
(2^{-256} chance)

Verifying Consistency w.r.t. $\mathbb{G}$

$\mathsf{MULT}([\phi], [k])$

Simplified :



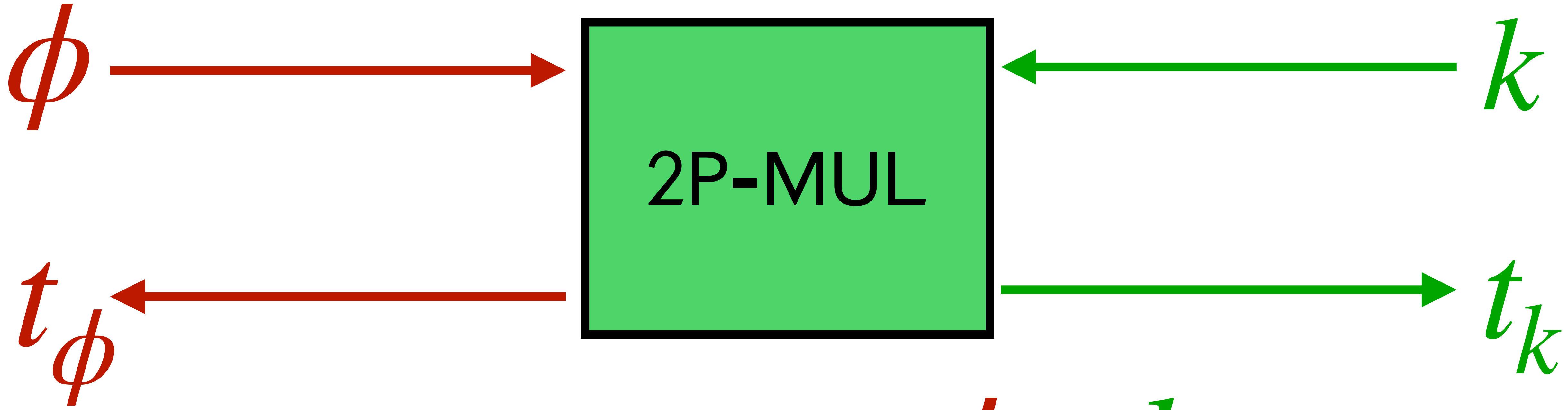
$$T_\phi = t_\phi \cdot G \qquad t_\phi + t_k = \phi \cdot k \qquad T_k = t_k \cdot G$$

Claimed $R^* = \Delta + k \cdot G$

Verifying Consistency w.r.t. $\mathbb{G}$

MULT($[\phi]$, $[k]$)

Simplified :



$$T_\phi = t_\phi \cdot G \qquad t_\phi + t_k = \phi \cdot k \qquad T_k = t_k \cdot G$$

Claimed $R^* = \Delta + k \cdot G$

$$T_k^* = \phi R^* - T_\phi$$

Verifying Consistency w.r.t. $\mathbb{G}$

MULT([ϕ], [k])

Simplified :

$$
\begin{tikzcd}
\phi \arrow[r] & \boxed{\text{2P-MUL}} \arrow[dl] \arrow[dr] & k \arrow[l] \\
t_\phi & & t_k
\end{tikzcd}
$$

$$T_\phi = t_\phi \cdot G \qquad t_\phi + t_k = \phi \cdot k \qquad T_k = t_k \cdot G$$

Claimed $R^* = \Delta + k \cdot G$

$$
\begin{aligned}
T_k^* &= \phi R^* - T_\phi \\
&= T_k + \phi\Delta
\end{aligned}
$$

Verifying Consistency w.r.t. $\mathbb{G}$

$\mathsf{MULT}([\phi], [k])$

Simplified :

$\phi \rightarrow$ 2P-MUL $\leftarrow k$

$t_\phi \leftarrow$ 2P-MUL $\rightarrow t_k$

$$t_\phi + t_k = \phi \cdot k$$

$$T_\phi = t_\phi \cdot G \qquad T_k = t_k \cdot G$$

Claimed $R^* = \Delta + k \cdot G$

$$T_k^* = \phi R^* - T_\phi$$
$$= T_k + \phi\Delta$$

No information about ϕ
$\Rightarrow$ distributed uniformly in $\mathbb{Z}_q$

Verifying Consistency w.r.t. $\mathbb{G}$

MULT($[\phi]$, $[k]$)

Simplified :

$$\phi \longrightarrow \boxed{\text{2P-MUL}} \longleftarrow k$$

$$t_\phi \longleftarrow \boxed{\text{2P-MUL}} \longrightarrow t_k$$

$$T_\phi = t_\phi \cdot G \qquad t_\phi + t_k = \phi \cdot k \qquad T_k = t_k \cdot G$$

Claimed $R^* = \Delta + k \cdot G$

$$T_k^* = \phi R^* - T_\phi$$
$$= T_k + \phi\Delta$$

No information about ϕ
$\Rightarrow$ distributed uniformly in $\mathbb{Z}_q$

Statistically unlikely to send correct T_k^*

Notes on Consistency Check

- Case 1: Inconsistent k^*—almost certainly fails
  Case 2: Consistent k— nothing about ϕ leaked
  $\Rightarrow$ ϕ is a MAC key, but also safe to (re)use in ECDSA tuple

- Costs 3 exponentiations, transmits single $\mathbb{G}$ element, one round
  All costs are superseded by 2P-MUL

- Exact same structure for $[\phi \mathsf{sk}]$ verification with pk

- Actual check: each party validates 2P-MUL inputs
  (i.e. shares of k, sk) used by every counterparty

3 Round ECDSA Signing

[This work]

Sample $[k]$

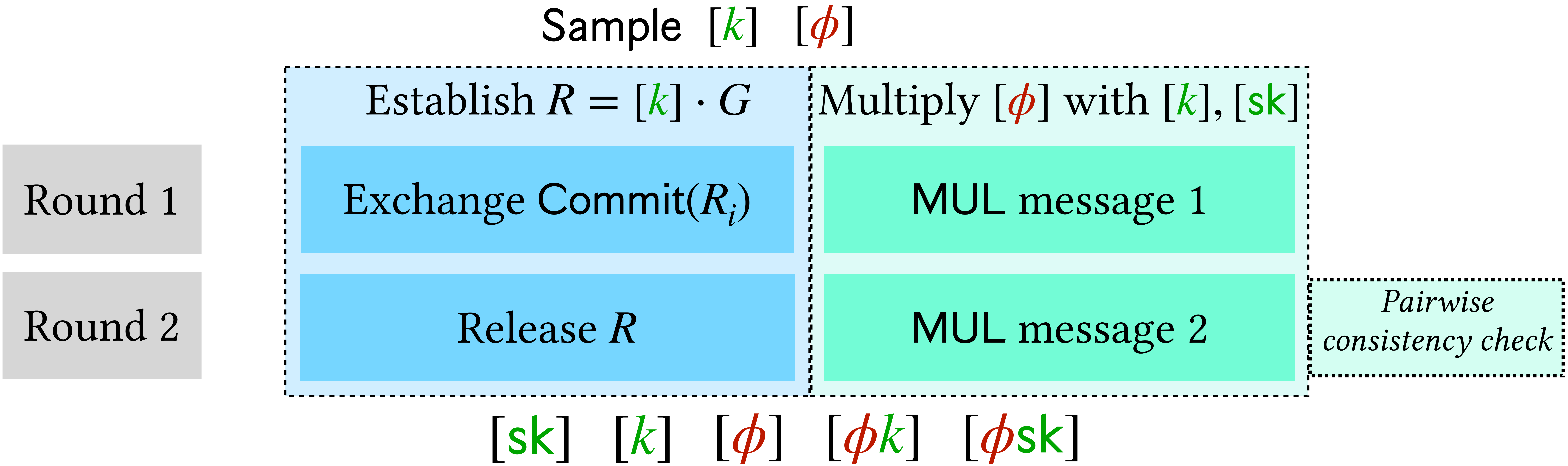
Establish $R = [k] \cdot G$

Round 1 — Exchange Commit(R_i)

Round 2 — Release R

3 Round ECDSA Signing

[This work]



3 Round ECDSA Signing

[This work]

Sample $[k]$ $[\phi]$

Establish $R = [k] \cdot G$

Multiply $[\phi]$ with $[k], [\mathsf{sk}]$

Round 1

Exchange $\mathsf{Commit}(R_i)$

MUL message 1

Round 2

Release R

MUL message 2

Pairwise consistency check

$[\mathsf{sk}]$ $[k]$ $[\phi]$ $[\phi k]$ $[\phi \mathsf{sk}]$

Round 3

Reveal $\alpha = e[\phi] + r_x[\phi\mathsf{sk}]$ and $\beta = [\phi k]$

Output $(R, \sigma = \alpha/\beta)$

Intro		How to distribute ECDSA			Tradeoffs	
MPC for Schnorr is easy	but not ECDSA	Rewriting ECDSA + 3 round protocol	ECDSA Tuples	2P-MUL + Consistency	OT vs AHE	Modes of operation

Instantiating Multiplication

- Secure n-party mult can be reduced to $2n$ instances of 2P-MUL
- 2P-MUL inherently requires public key crypto
- Broadly two approaches:
  - Additively Homomorphic Encryption (low bandwidth, high computation)
  - Oblivious Transfer (low computation, high bandwidth)

2P-MUL from Additively Homomorphic Encryption

- Additive Homomorphism: $\alpha \cdot \mathsf{Enc}(x) + \mathsf{Enc}(\beta) = \mathsf{Enc}(\alpha x + \beta)$

  [Gilboa 99]: Conceptually simple protocol for MUL from AHE
  [CGGMP 20]: Hardened for active security through ZK proofs

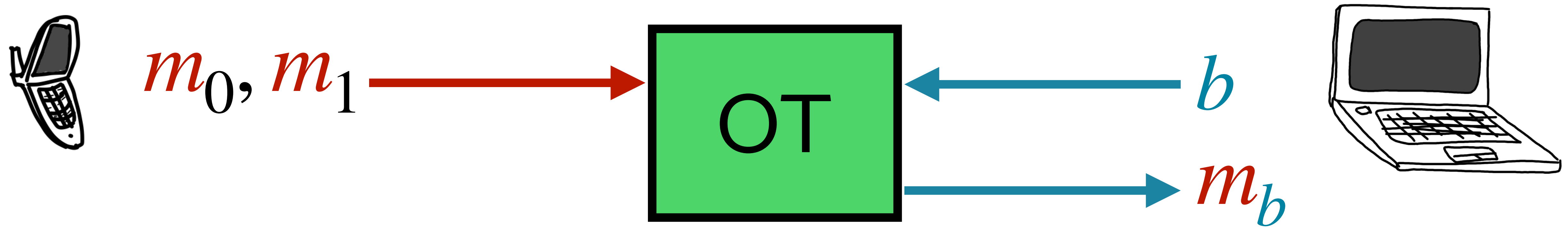
- Instantiations from factoring based cryptography (e.g. [Paillier 99]) and class groups [Castagnos Laguillaumie 15]

- <u>Advantages</u>: Parties exchange (relatively) compact ciphertexts

- <u>Downsides</u>:
  — Ciphertext operations are heavy (2 orders of magnitude slower than EC)
  — Seem to require ZK proofs to prevent misuse

2P-MUL from Oblivious Transfer

- Oblivious Transfer (OT):



[Gilboa 99]: Elegant protocol for MUL from OT
[DKLs 18,19, HMRT 22]: active security by randomized encoding+statistical checks

- Instantiable with ECDSA curve (think DH key exchange)

- Advantages: By OT Extension [IKNP03, Roy22] public key operations can be moved to one-time key generation phase, so only hashes when signing (1 order of magnitude slower than single party signing)

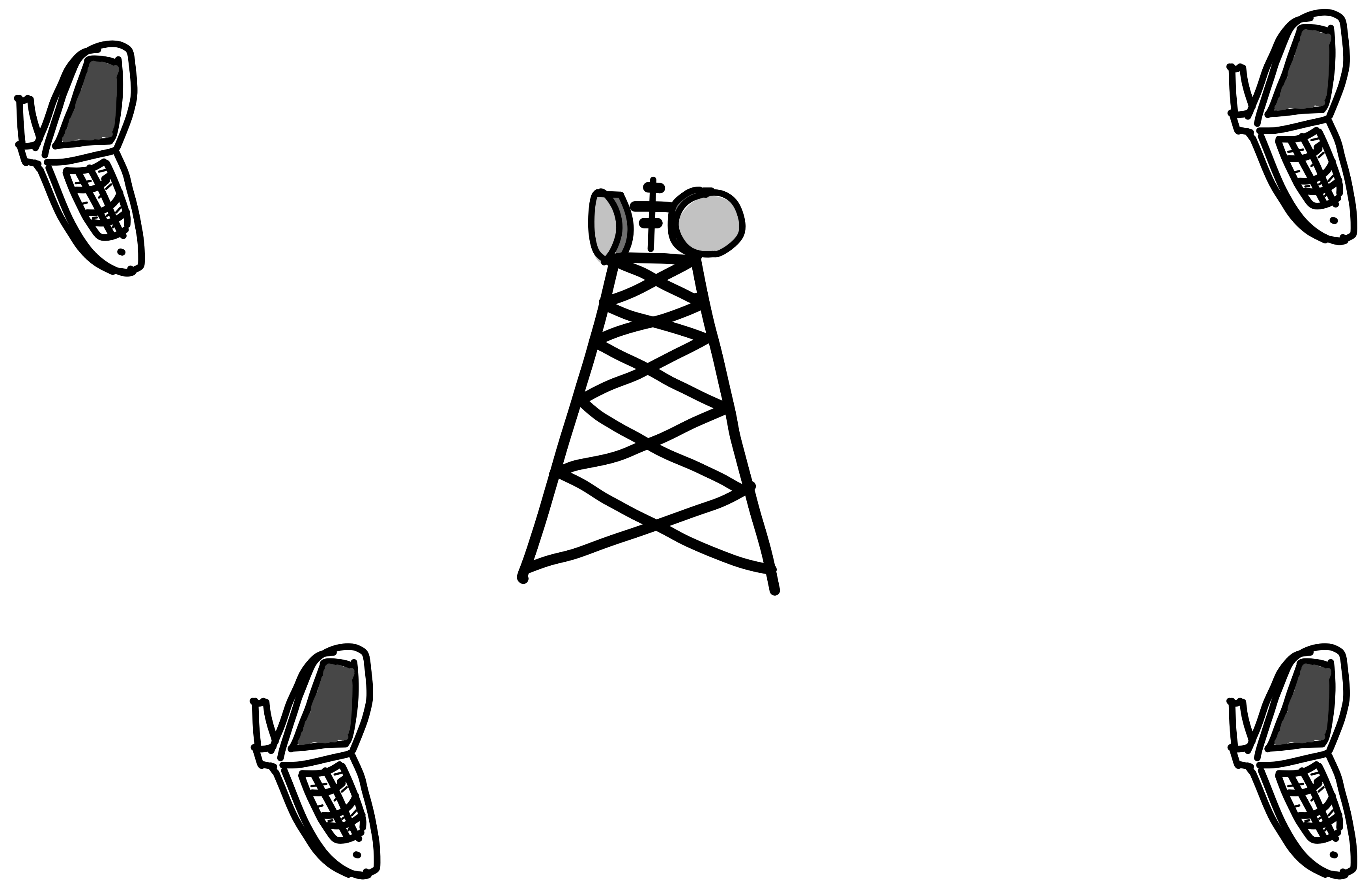
- Downsides: ~1000 OTs/sig, each transmits two $\mathbb{Z}_q$ elements

2P-MUL: AHE vs OT

- Tradeoff to make: Computation vs. Bandwidth during signing time
- Rough costs with 256-bit curve, for each additional party (computation aggregated across [Gavenda 21, XAXYC 21, BMP 22]):

	Bandwidth	Computation
OT [DKLs 23]	60 KB	Few milliseconds
Paillier [CGGMP 20]	15 KB	Hundreds of milliseconds
Paillier [GG 18]	7 KB	Hundreds of milliseconds
Class Groups [CCLST20, YCX21]	4.5 KB	> 1 second

Is communication the bottleneck?



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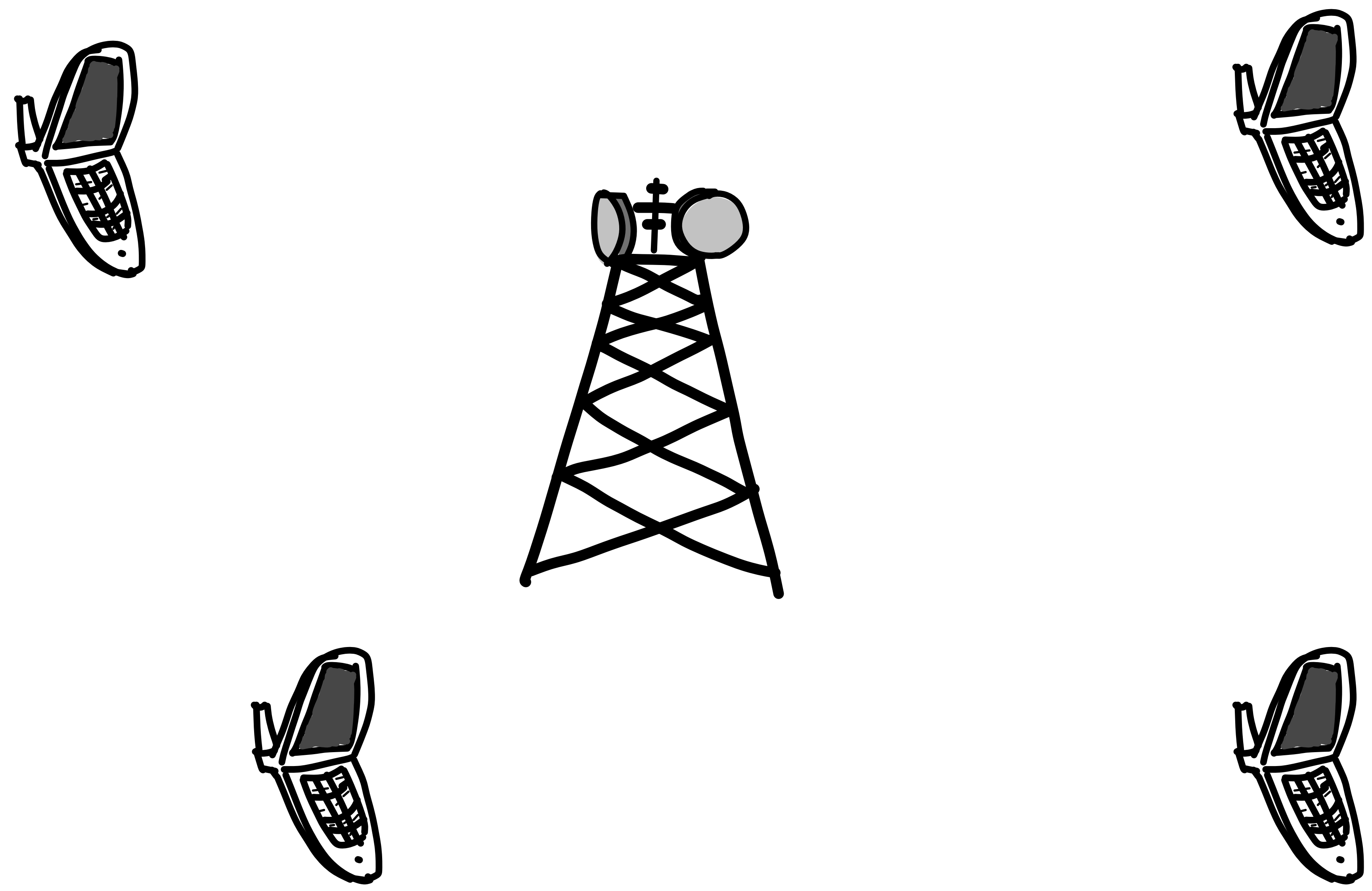

- **Mobile applications (human-initiated)**:

Is communication the bottleneck?

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  - eg. t=4, ~2Mbits transmitted per party

Is communication the bottleneck?

- **Mobile applications (human-initiated)**:
  - eg. t=4, ~2Mbits transmitted per party
  - Well within LTE envelope for responsivity

Example 1: Mobile Wallet

Multiplier: OT-based
Parties: 4
Curve: 256-bit

2 Mbits

sent per party

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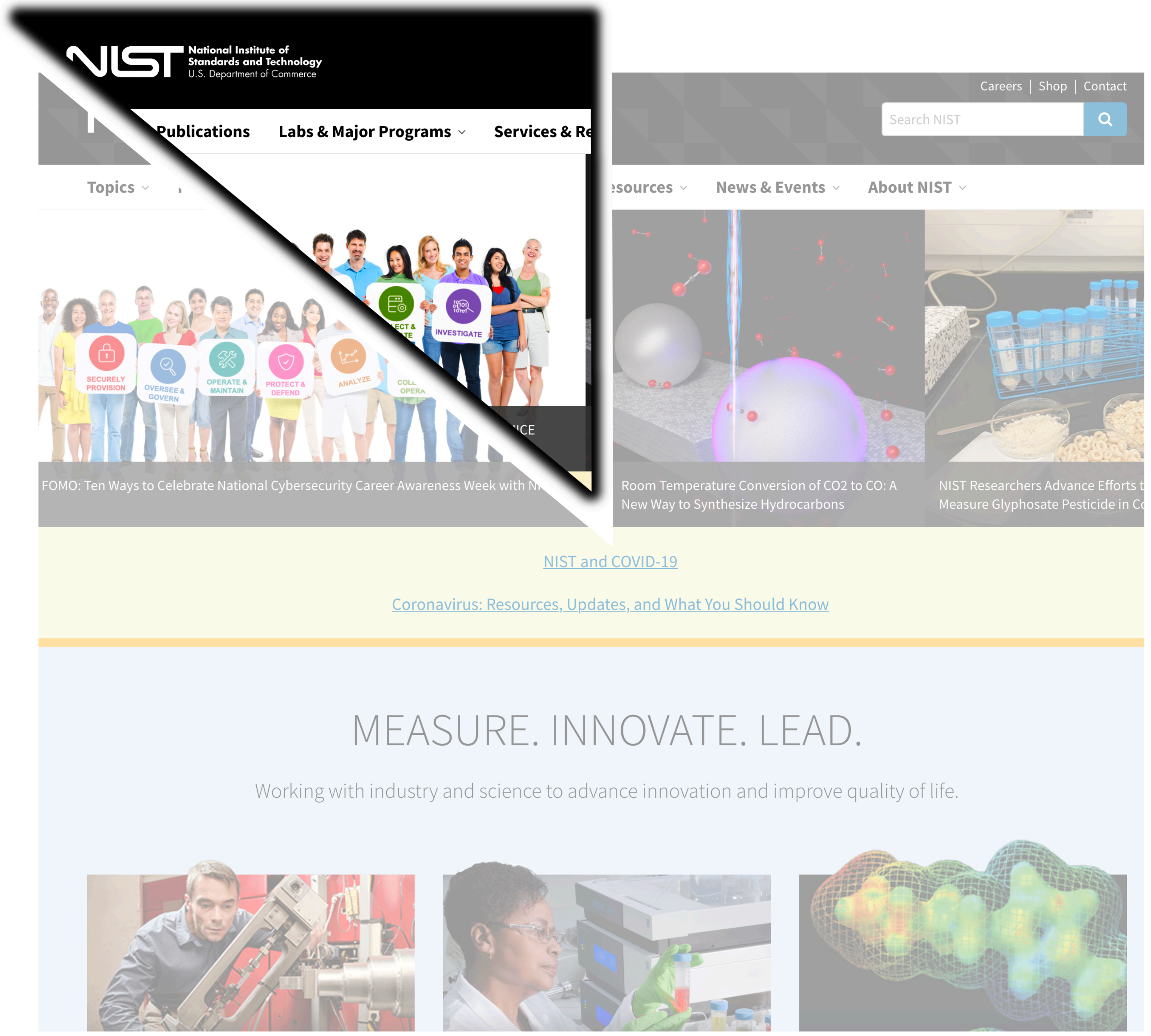
$\approx$



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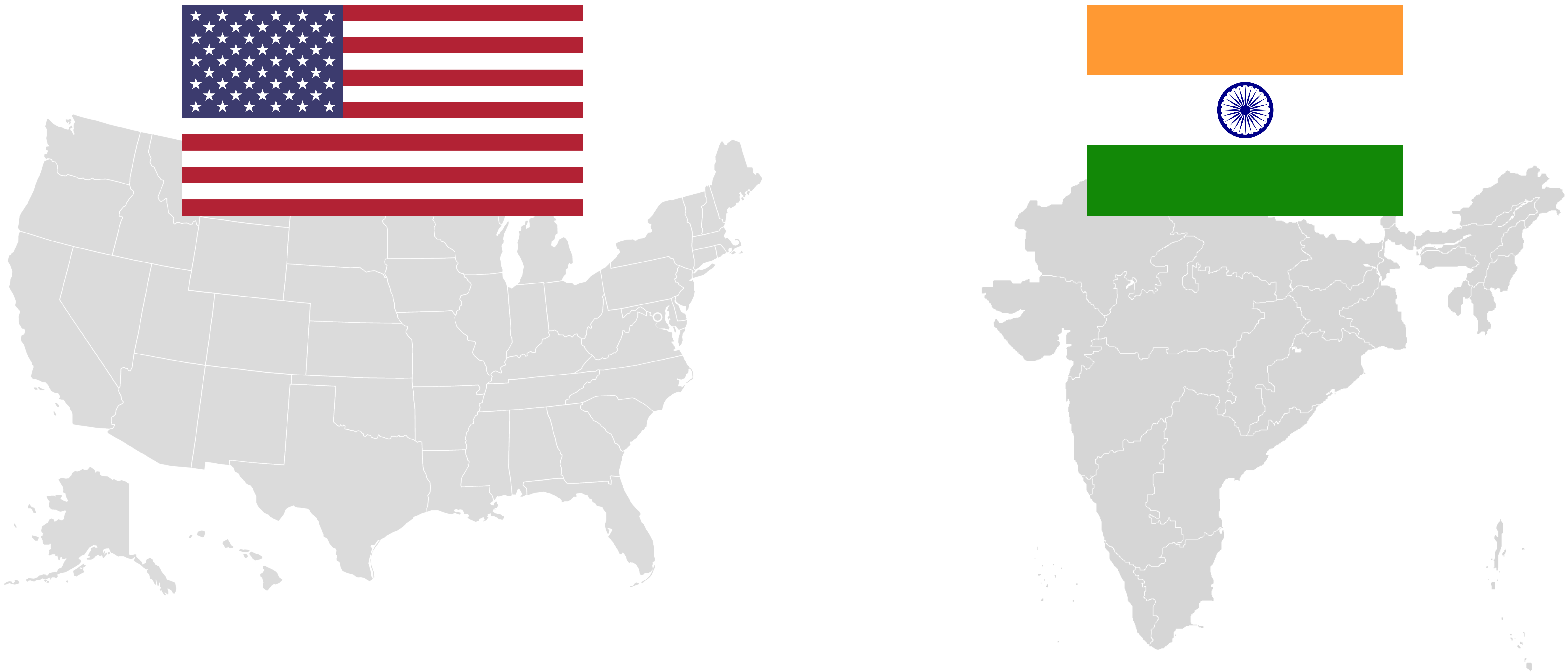

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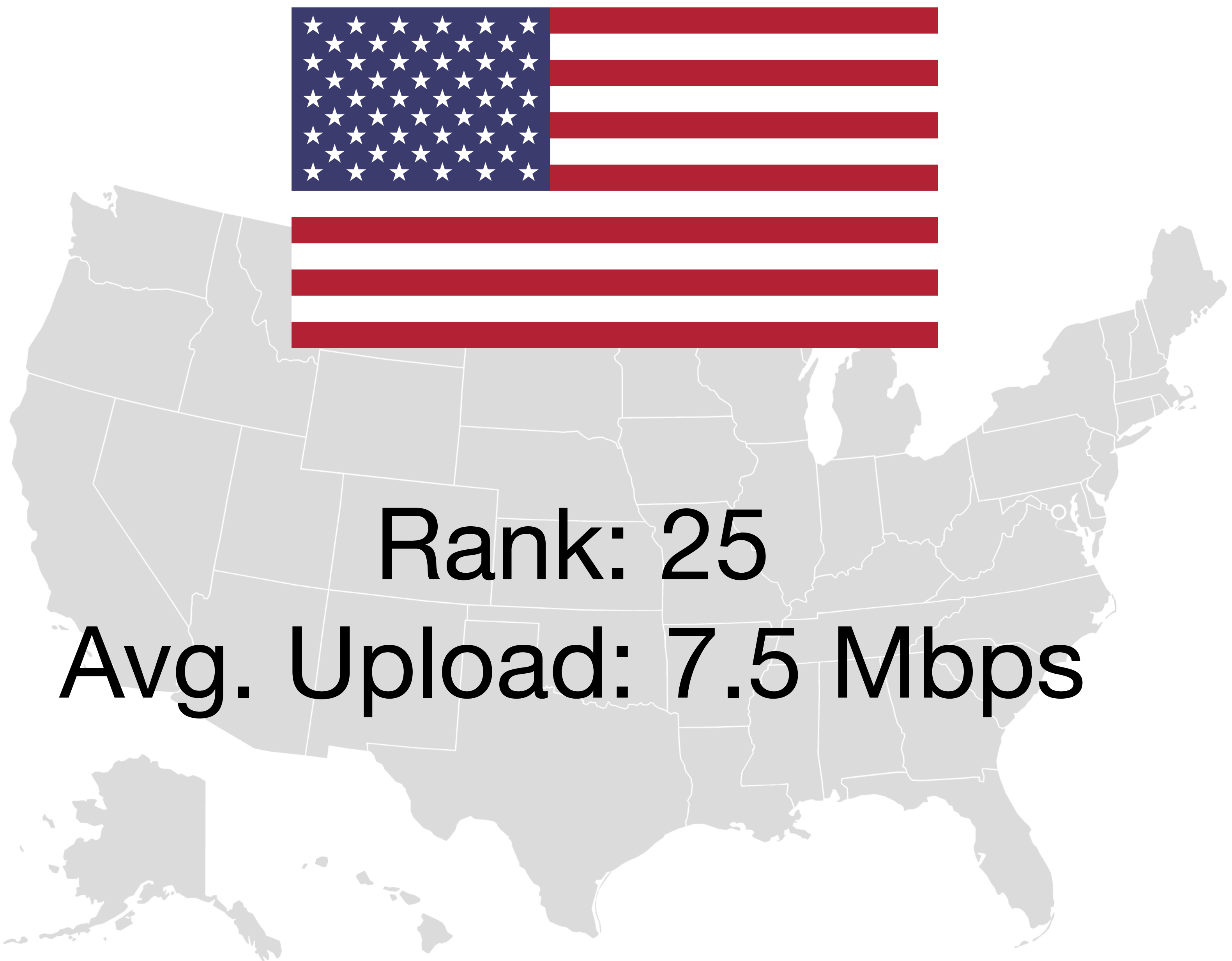

Rank: 25
Avg. Upload: 7.5 Mbps

source: opensignal

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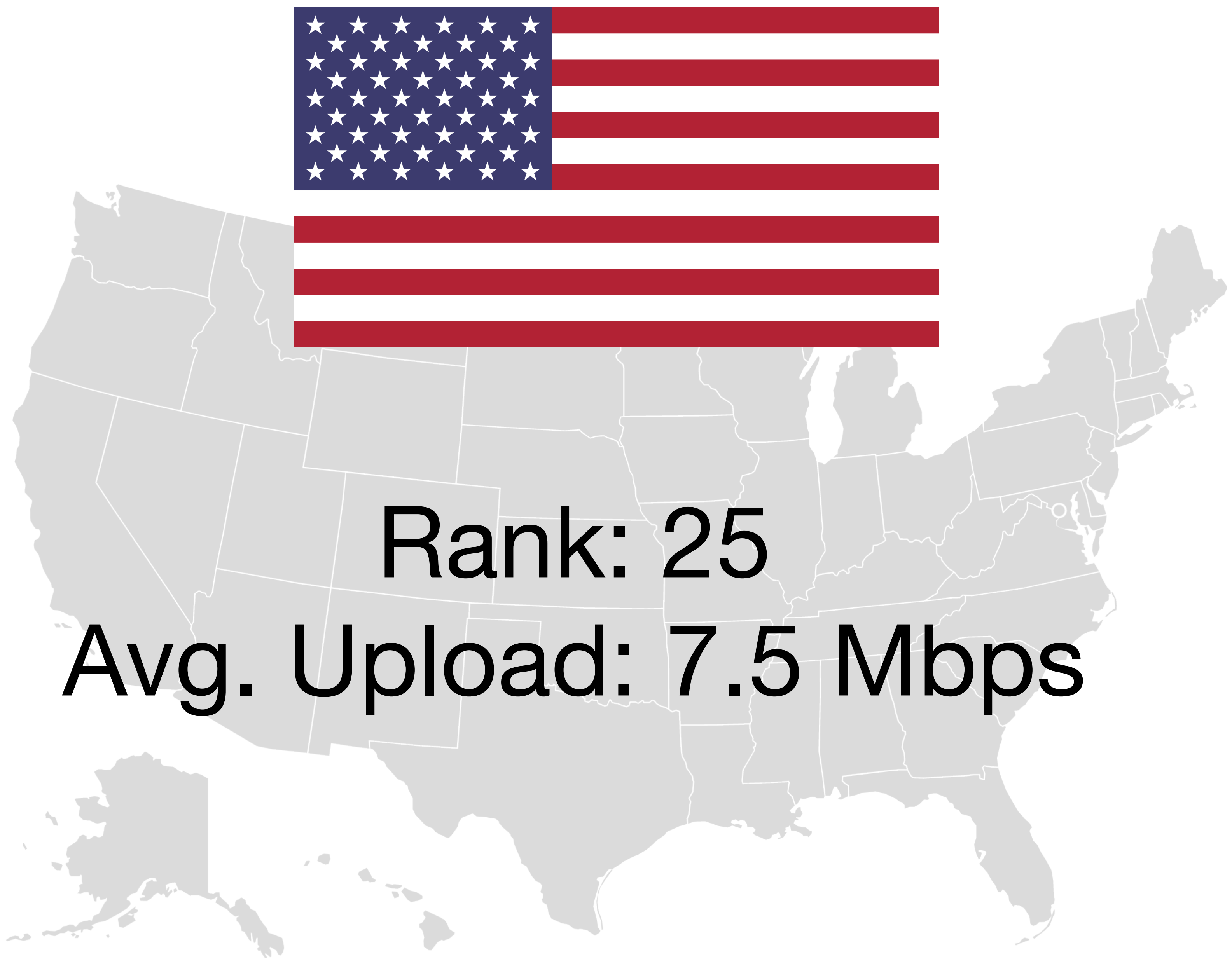

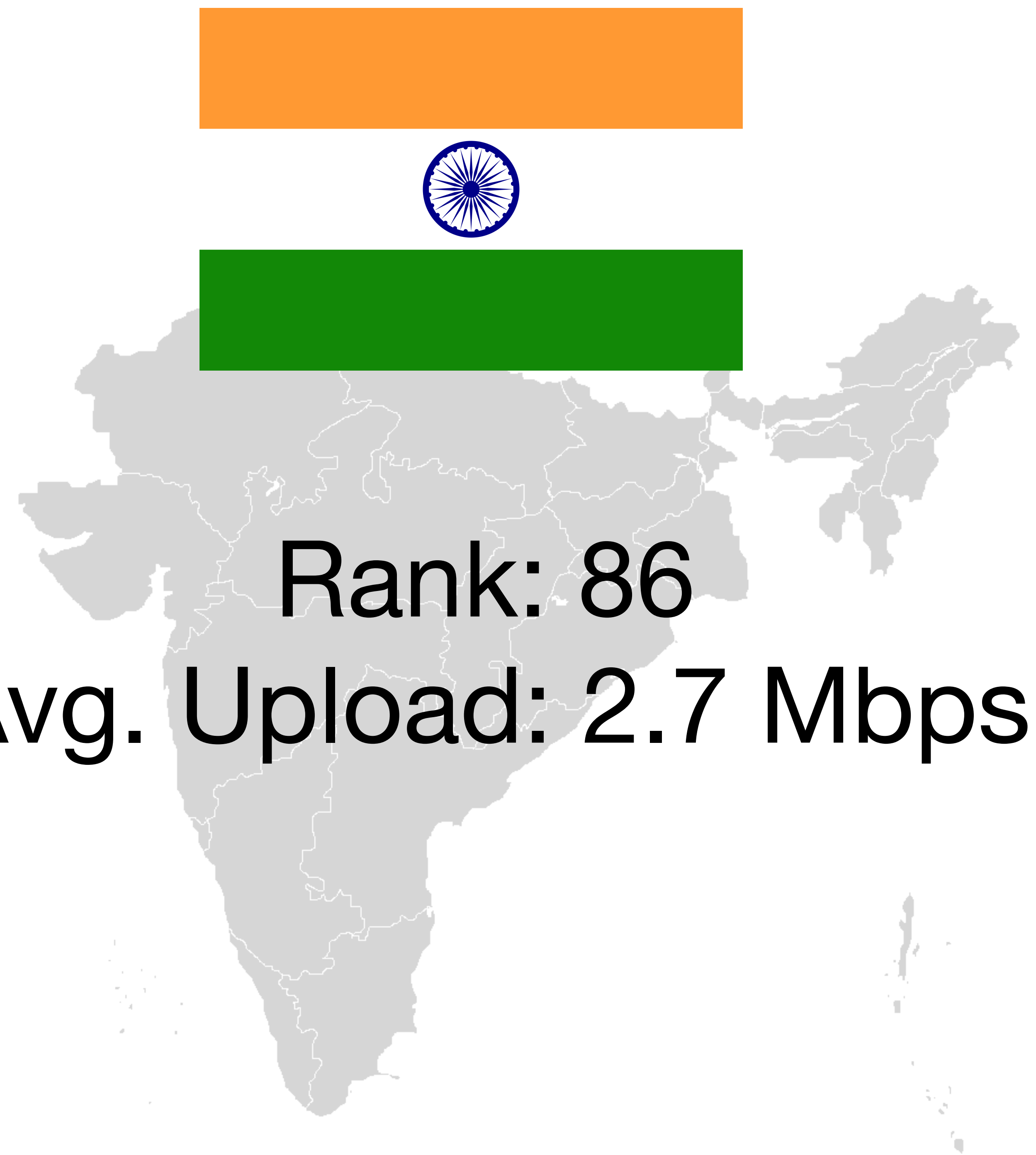

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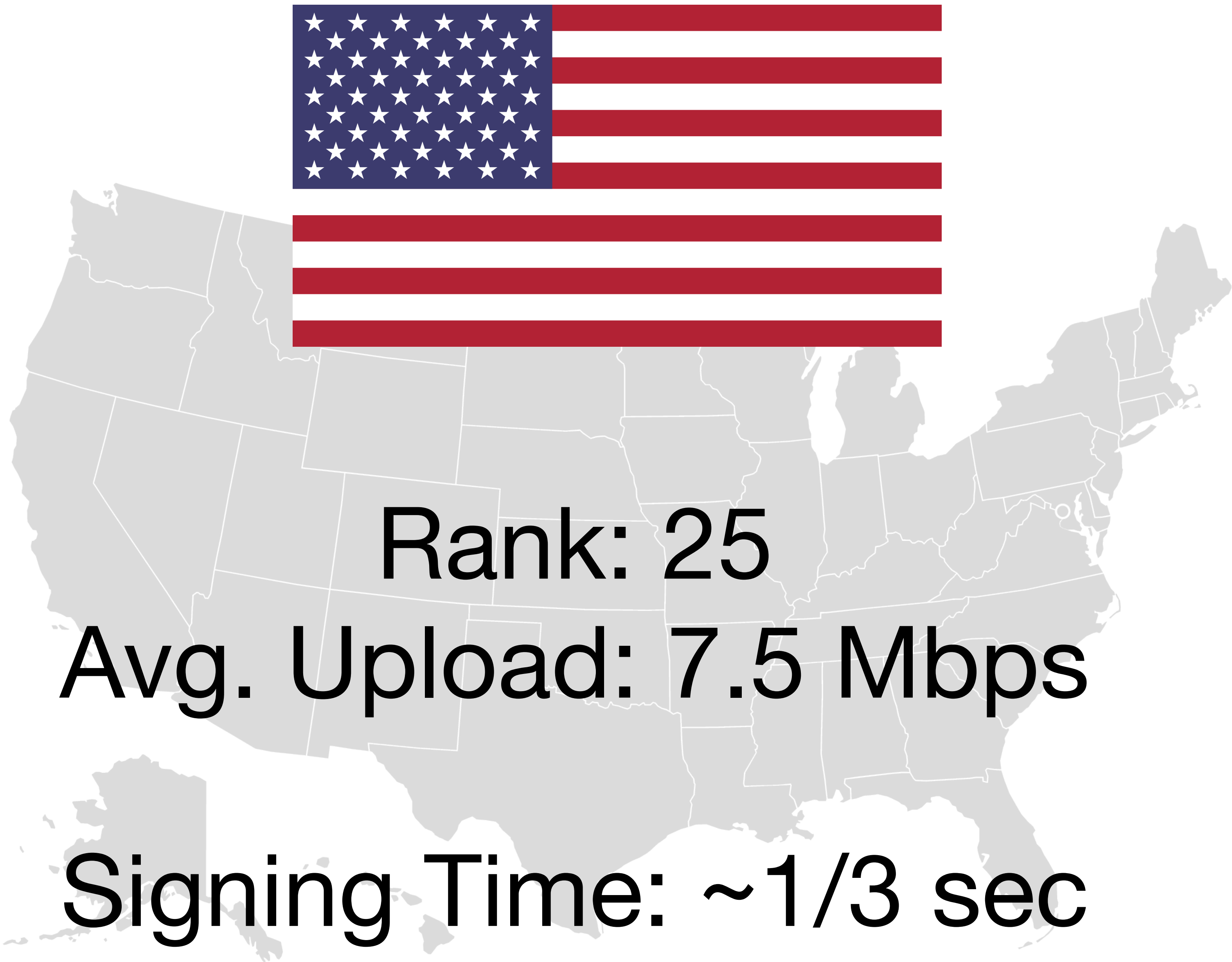

Rank: 25
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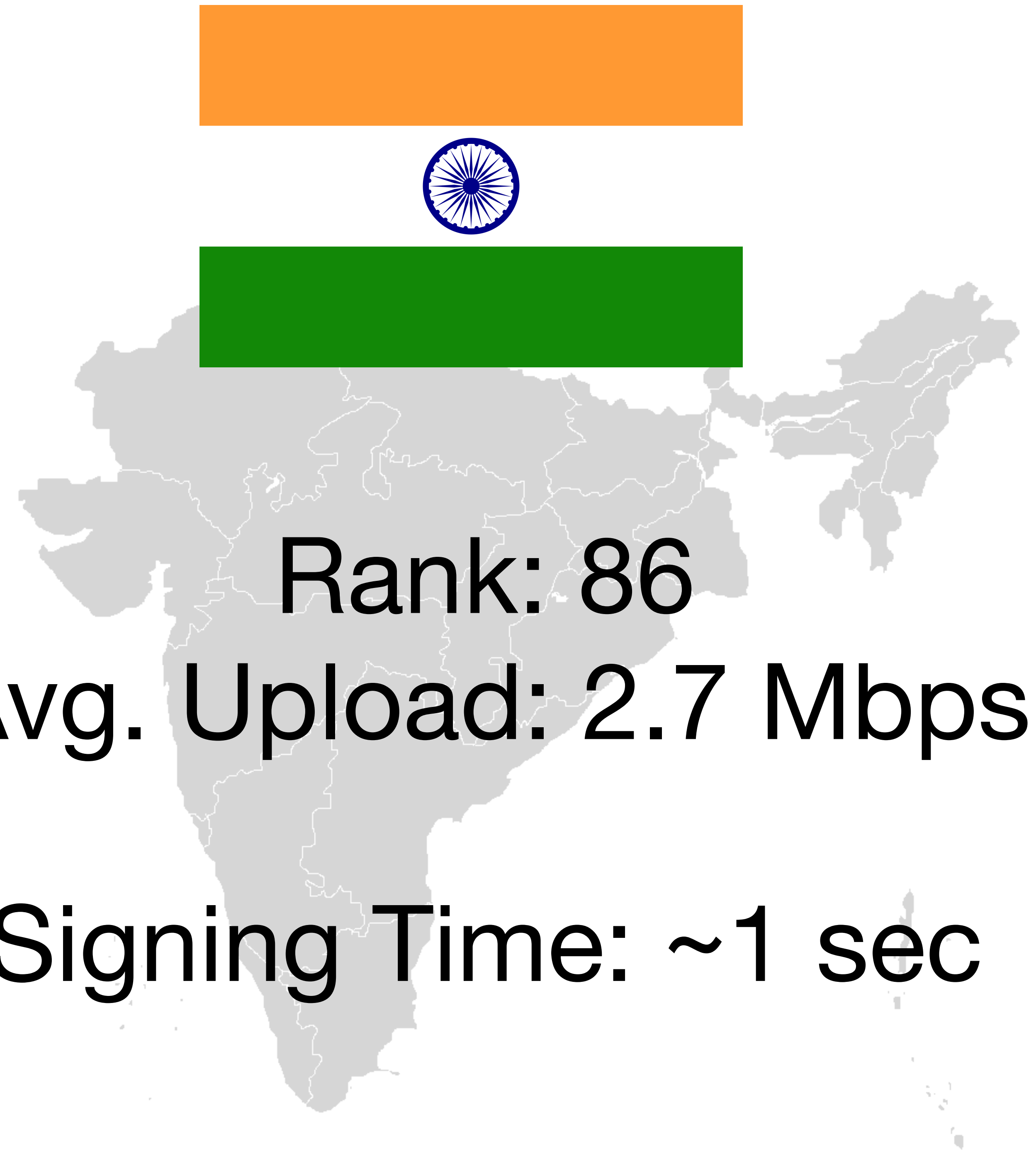
Signing Time: ~1/3 sec



Rank: 86
Avg. Upload: 2.7 Mbps

Signing Time: ~1 sec

source: opensignal

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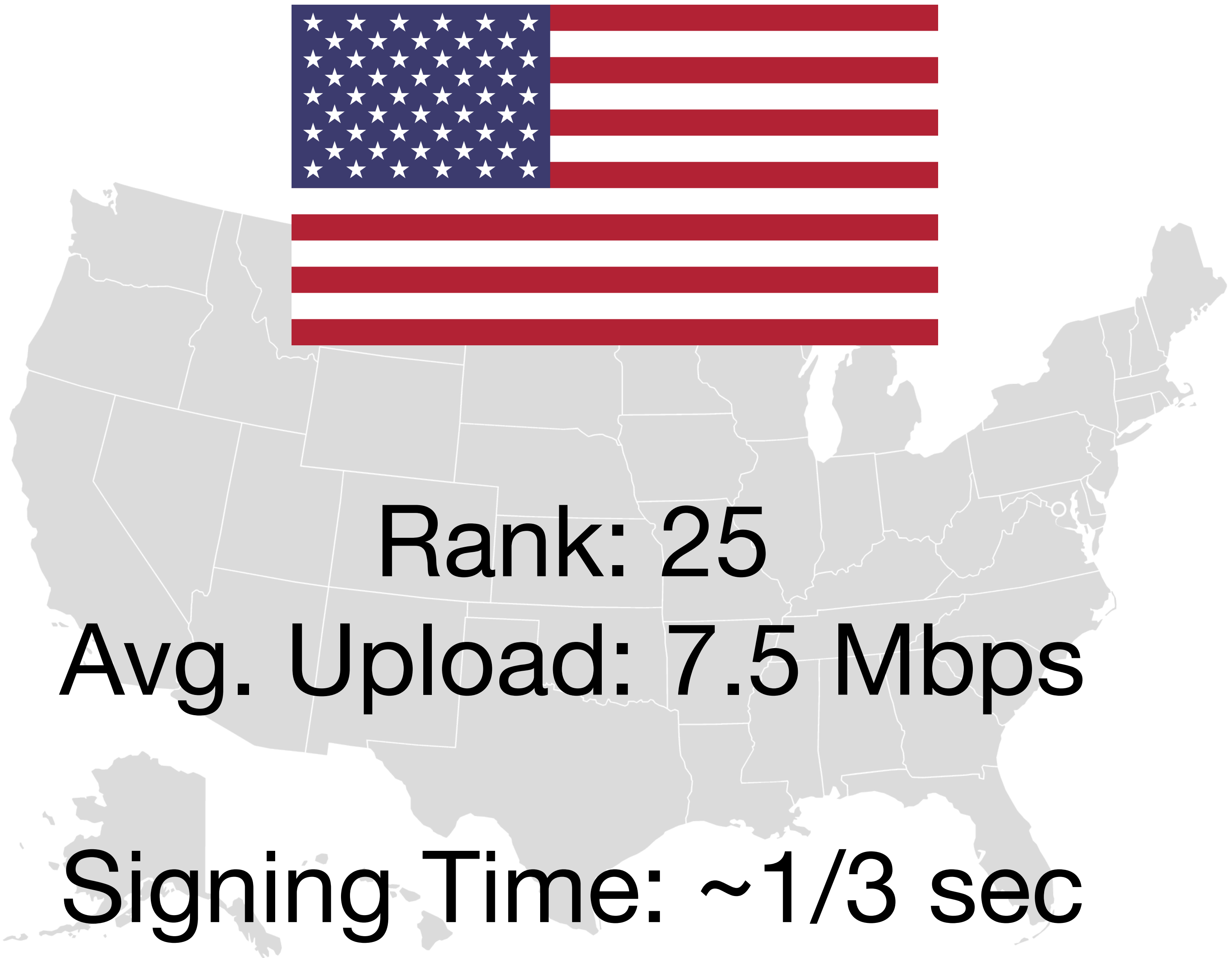

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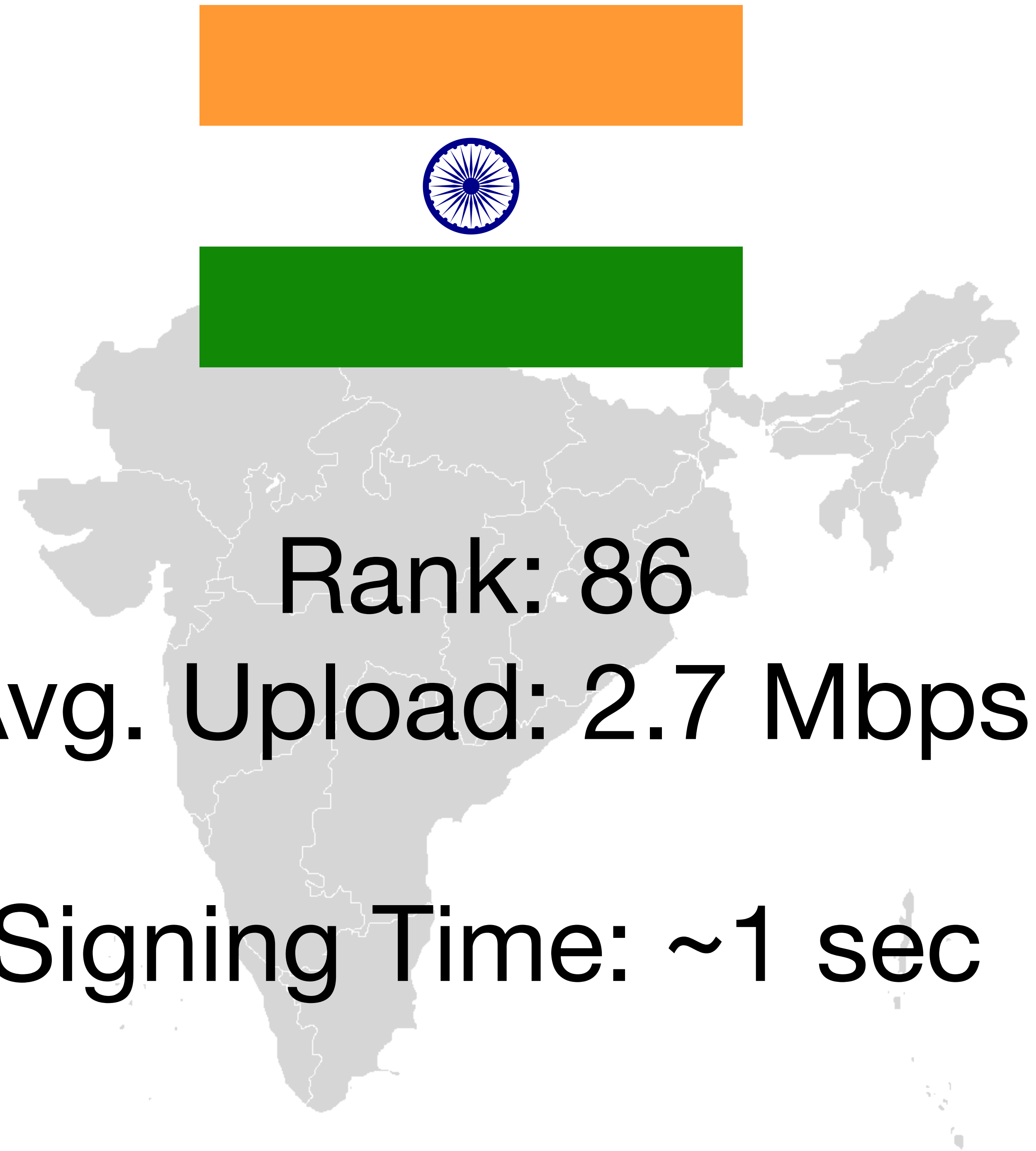

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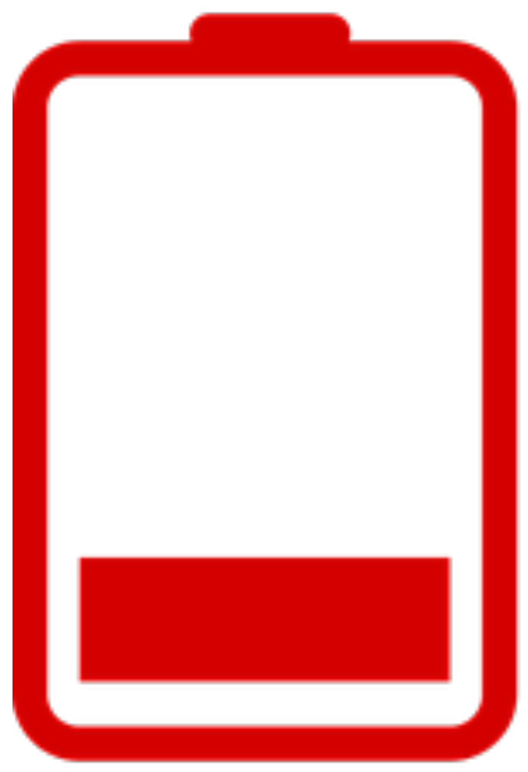
Avg. Upload: 2.7 Mbps

Signing Time: ~1 sec

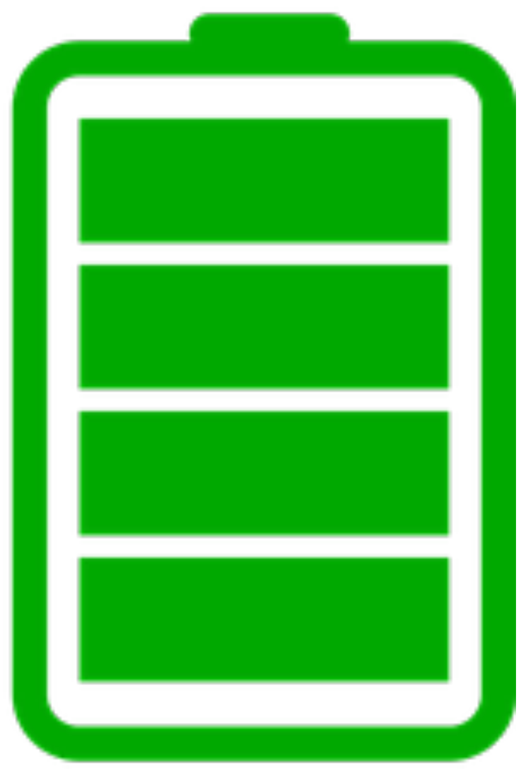
Similar to computation time for Paillier
on powerful hardware!

source: opensignal

On the Other Hand

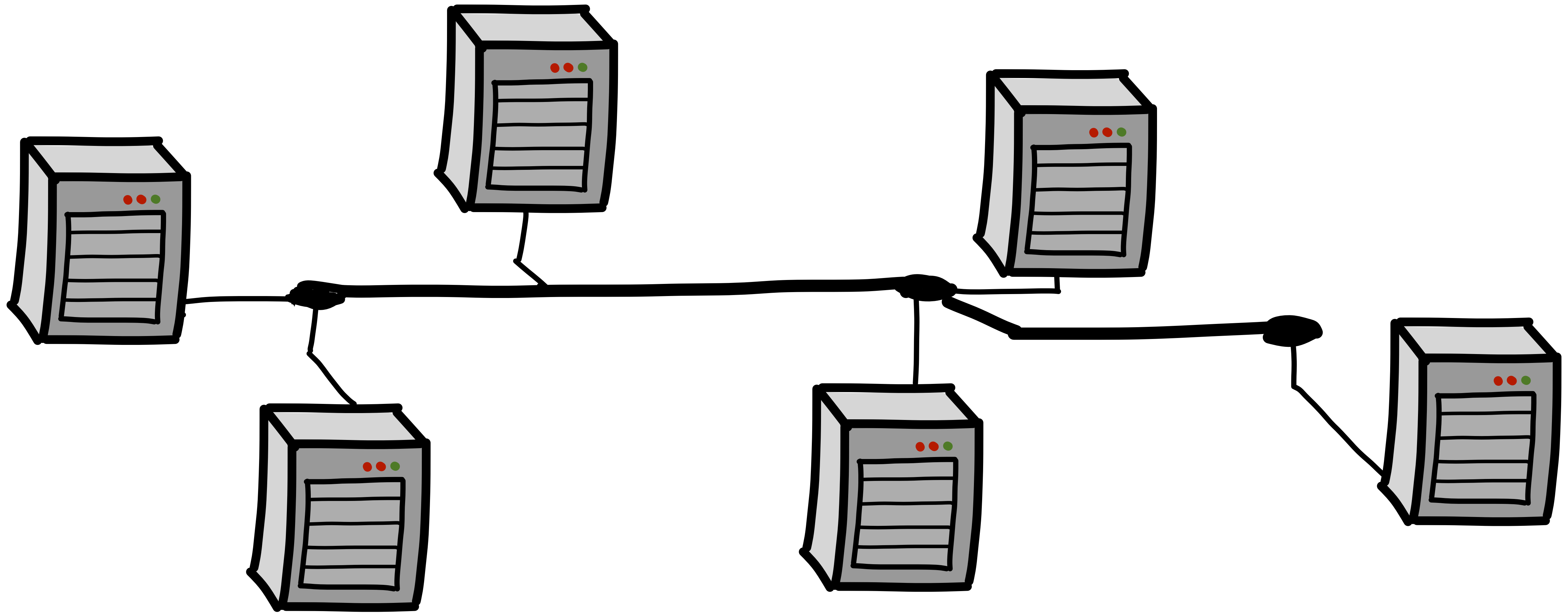


Paillier + ZK



OT

Is communication the bottleneck?



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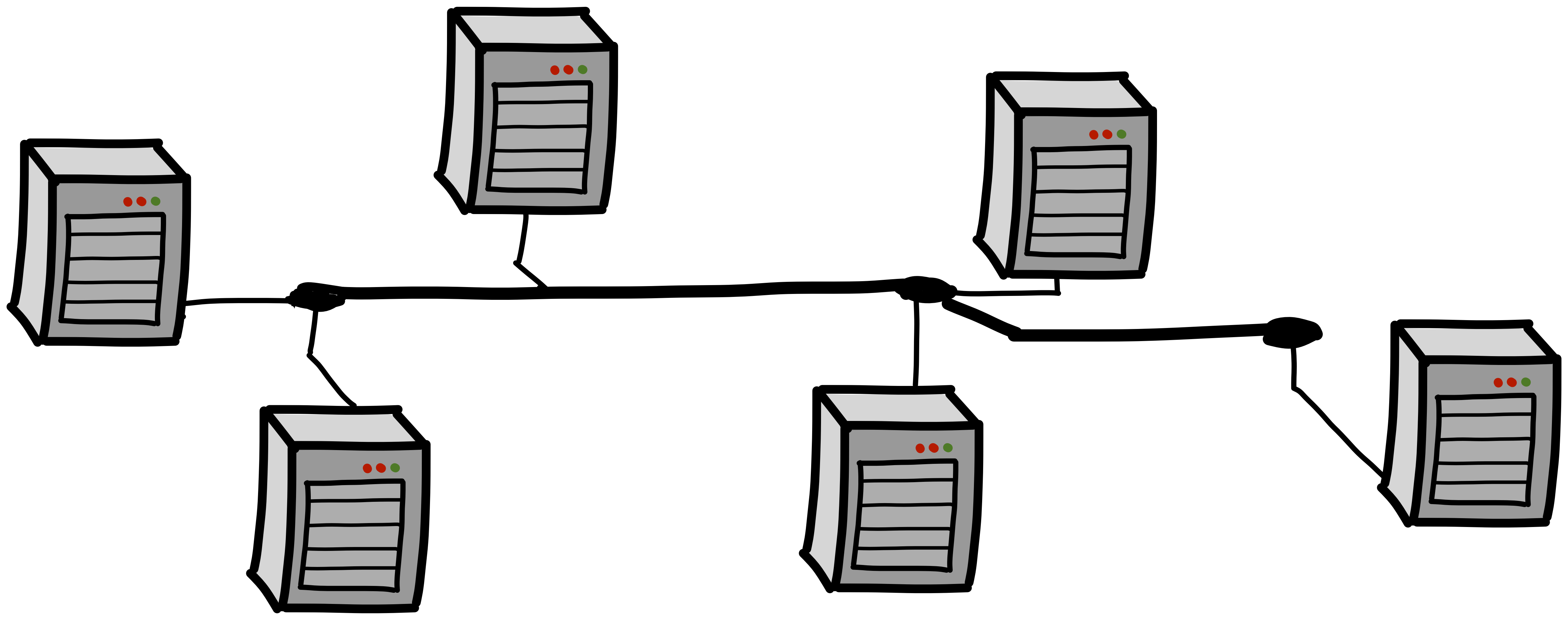
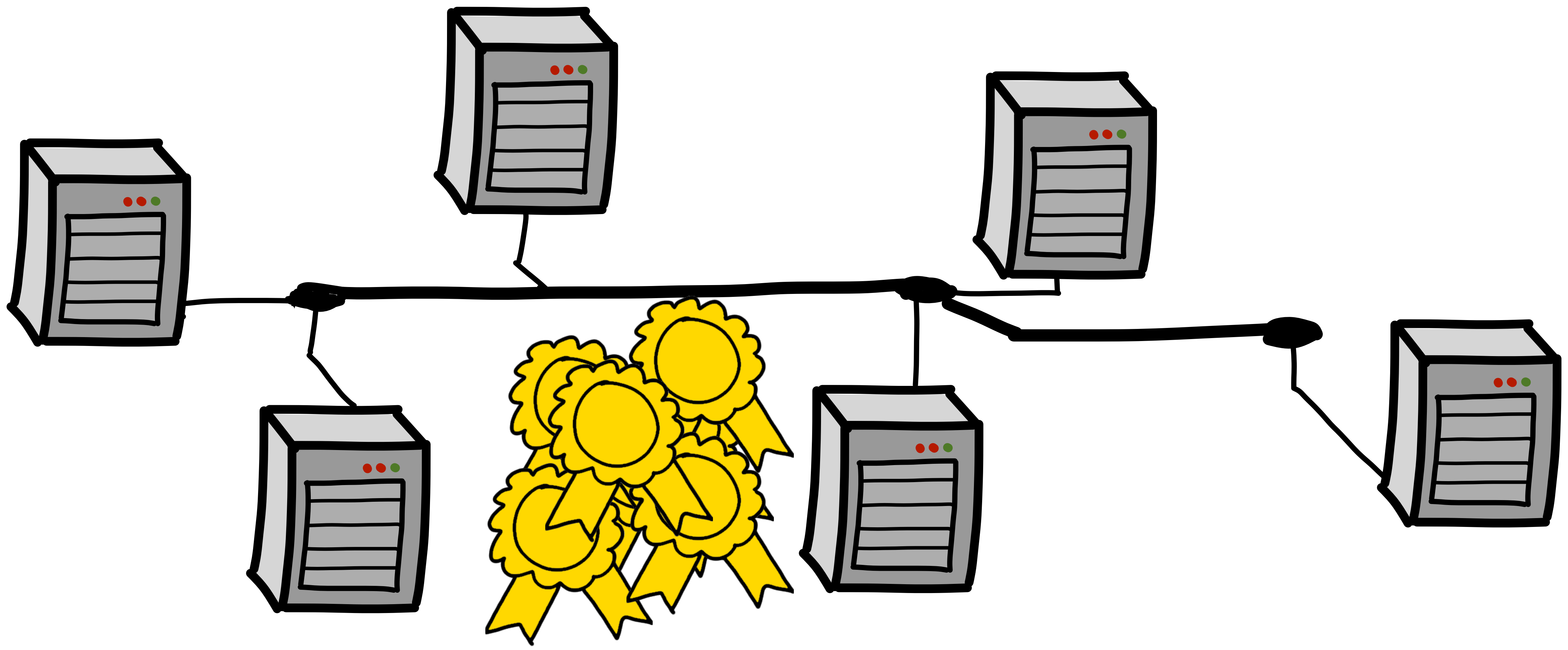


- **Large-scale automated distributed signing:**

Is communication the bottleneck?



- **Large-scale automated distributed signing:**

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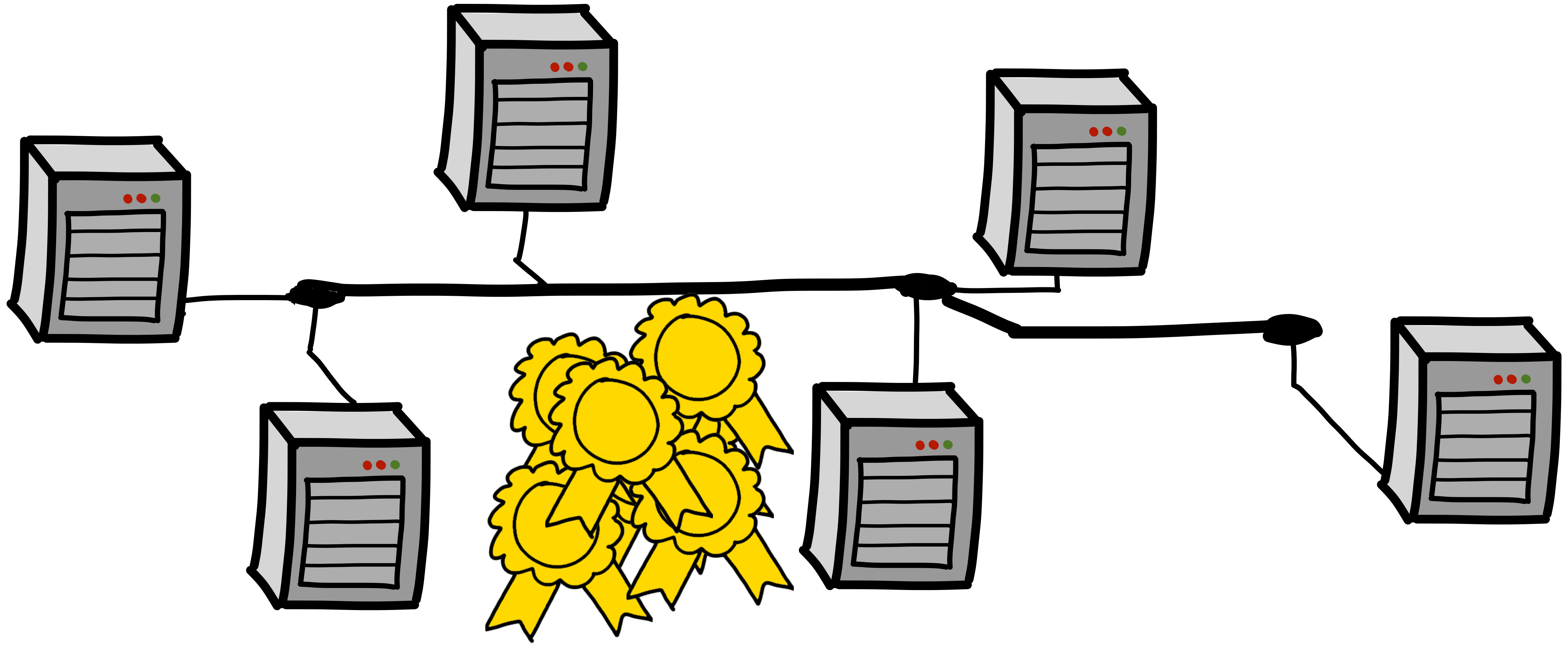
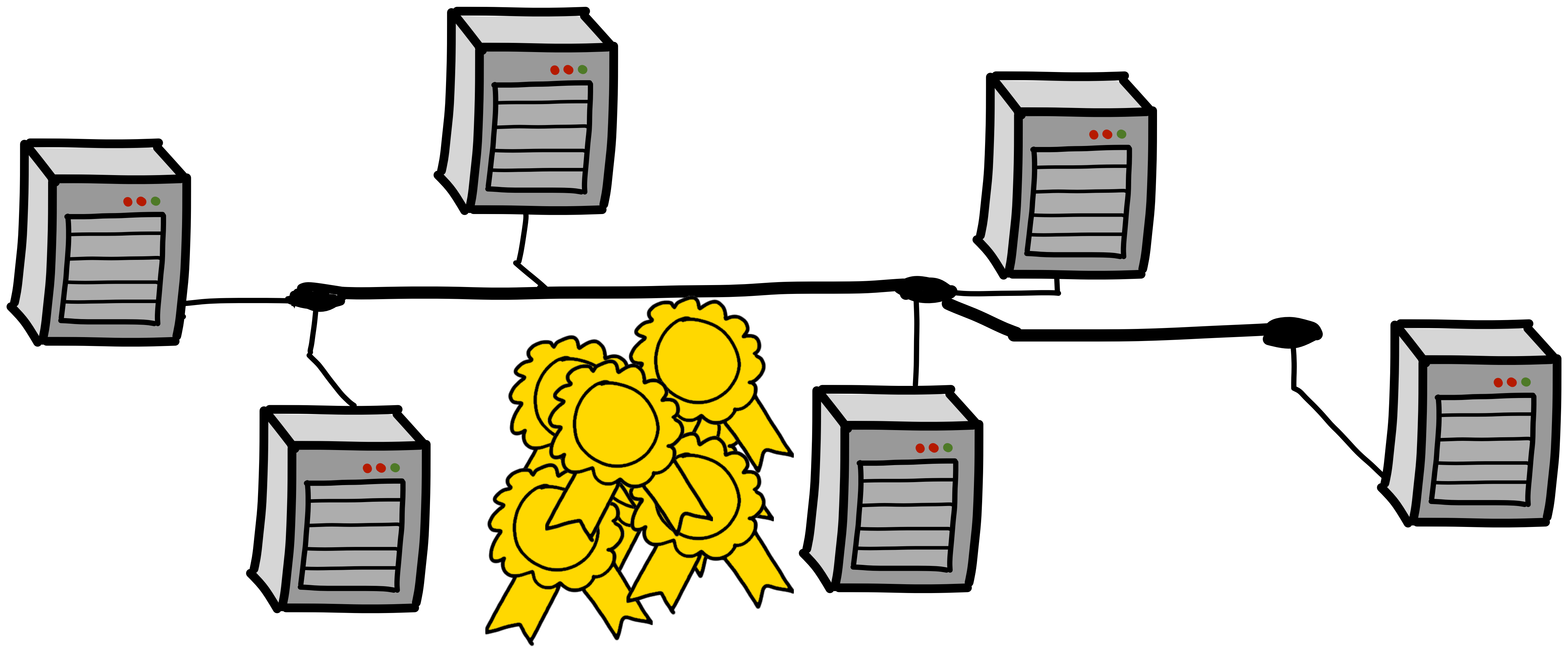


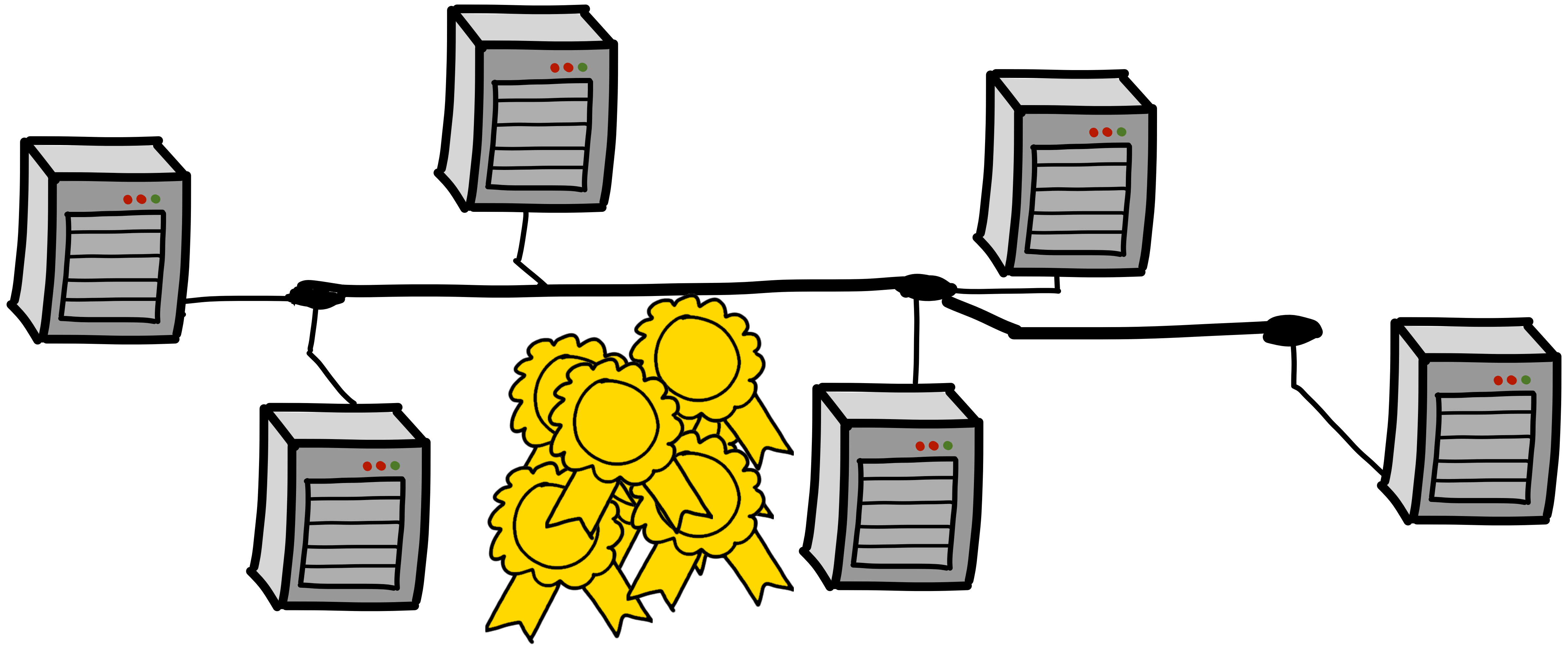
- **Large-scale automated distributed signing:**
  - Threshold 2: 3.8ms/sig <= ~263 sig/second

Is communication the bottleneck?



- **Large-scale automated distributed signing:**
  - Threshold 2: 3.8ms/sig <= ~263 sig/second
  - Threshold 20: 31.6ms/sig <= ~31 sig/second

Is communication the bottleneck?



- **Large-scale automated distributed signing:**
  - Threshold 2: 3.8ms/sig <= ~263 sig/second
  - Threshold 20: 31.6ms/sig <= ~31 sig/second
- Neither setting saturates a gigabit connection

Example 2: Datacenter Signing

How much bandwidth to be CPU bound?
(including preprocessing)

2 Parties
~250 sigs/second

256 Parties
~3 sigs/second

using GCP n1-highcpu nodes

Example 2: Datacenter Signing

How much bandwidth to be CPU bound?
(including preprocessing)

2 Parties
~250 sigs/second

Each party sends:
~700 Kbits per sig

256 Parties
~3 sigs/second

Each party sends:
~185 Mbits per sig

using GCP n1-highcpu nodes

Example 2: Datacenter Signing

How much bandwidth to be CPU bound?
(including preprocessing)

2 Parties
~250 sigs/second

Each party sends:
~700 Kbits per sig

Bandwidth required:
~180 Mbps symmetric

256 Parties
~3 sigs/second

Each party sends:
~185 Mbits per sig

Bandwidth required:
~555 Mbps symmetric

using GCP n1-highcpu nodes

Non-interactive Online Signing

Most Threshold ECDSA protocols have this format ([DOKSS20, CGGMP20] were the first to “use” it)

$\mathsf{Sign}([\mathsf{sk}], m)$

Round 1

⋮

Round $r - 1$

Only this round needs m

Round r

Non-interactive Online Signing

Sign([sk],)

Most Threshold ECDSA protocols have this format ([DOKSS20, CGGMP20] were the first to "use" it)

Round 1

⋮

Round $r - 1$

m is now available

Only this round needs m

Round r

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⋮

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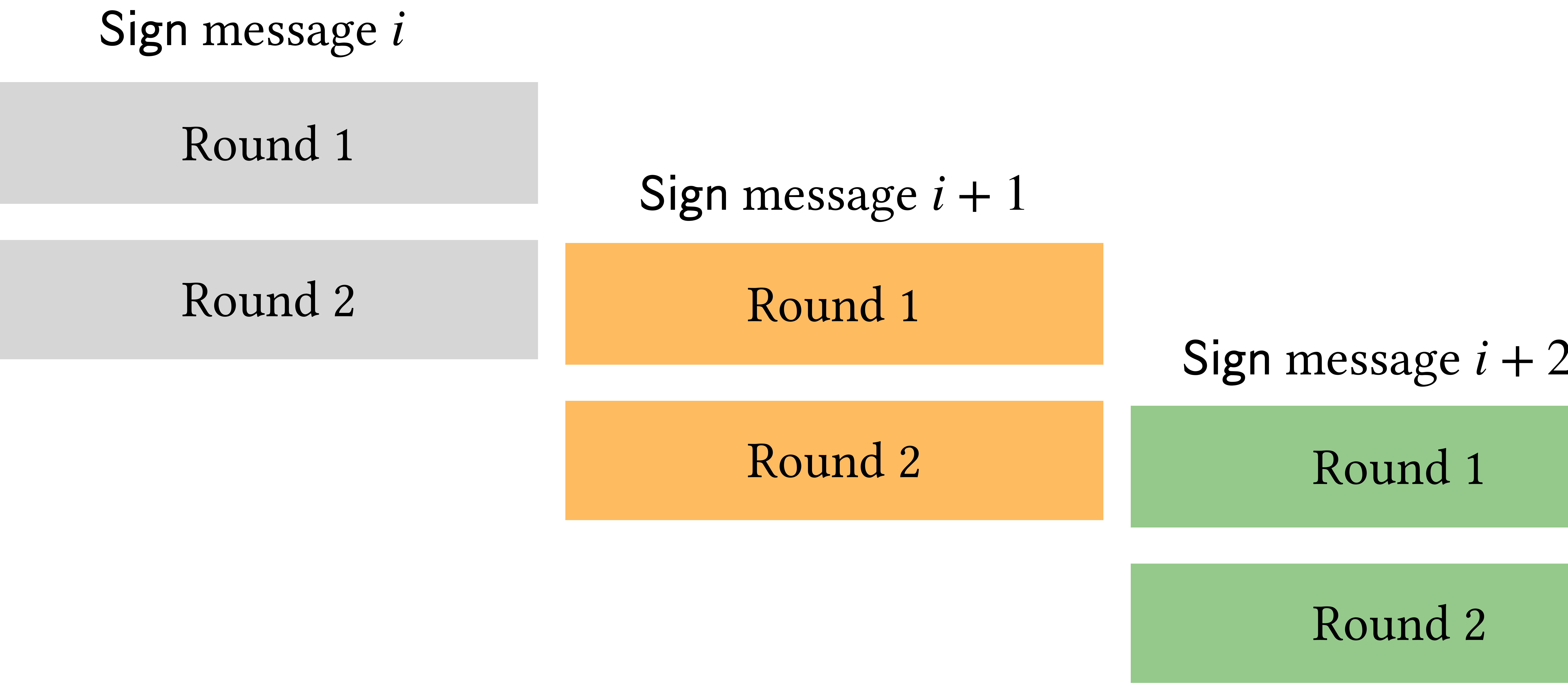
Round r

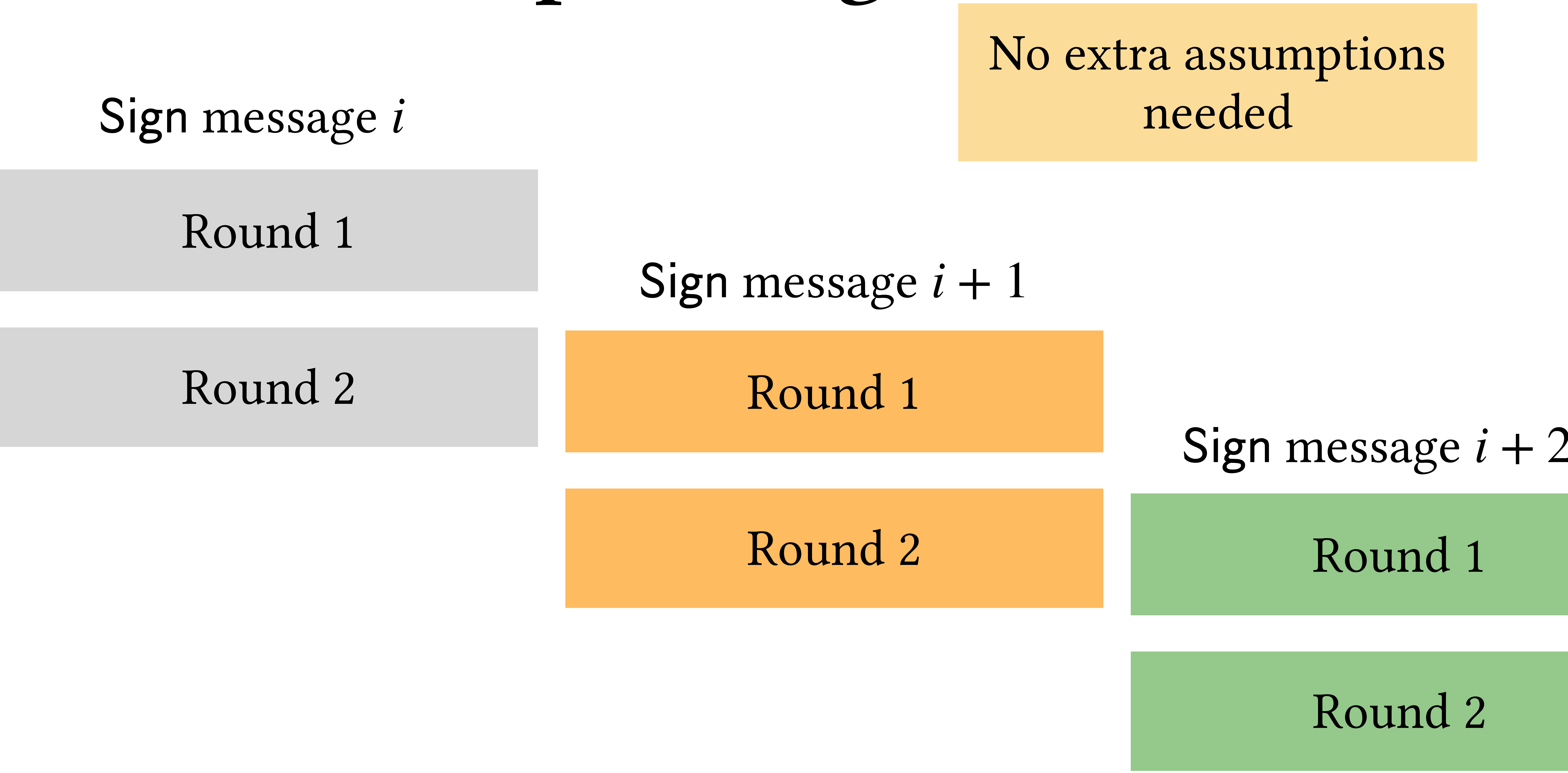
<u>Caveat</u>

Requires a stronger assumption on ECDSA, which is proven to hold in the GGM [Groth Shoup 22]

Pipelining


Sign message i
Round 1
Round 2
Sign message $i + 1$
Round 1
Round 2
Sign message $i + 2$
Round 1
Round 2

Pipelining


No extra assumptions needed
Sign message i
Round 1
Round 2
Sign message $i+1$
Round 1
Round 2
Sign message $i+2$
Round 1
Round 2

Pipelining

No extra assumptions needed

Sign message i

Round 1

Round 2

Sign message $i + 1$

Round 1

Round 2

Sign message $i + 2$

Round 1

Round 2

Saves a round on average

Intro		How to distribute ECDSA			Tradeoffs	
MPC for Schnorr is easy	but not ECDSA	Rewriting ECDSA + 3 round protocol	ECDSA Tuples	2P-MUL + Consistency	OT vs AHE	Modes of operation

In Conclusion

- Threshold ECDSA in Three Rounds: Now matches Schnorr
- Enabled by well-chosen correlation + simple new consistency check
- Blackbox use of UC 2-round 2P-MUL
  NOTE: OT-based protocols satisfy UC, but AHE is more complicated
- No ZK proofs during signing: light protocol and straightforward UC analysis

dkls.info

Thanks!

Thanks **Eysa Lee** for