Sometimes You Can't Distribute Random-Oracle-Based Proofs

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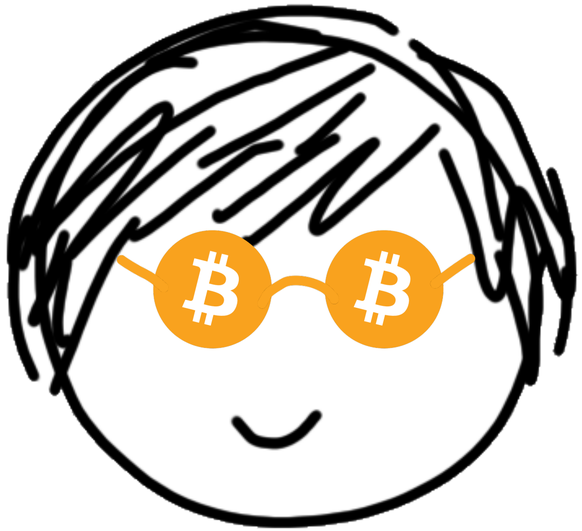
TECHNION Israel Institute of Technology

AARHUS UNIVERSITY

BROWN

Ballad of ₿itcoin Bob



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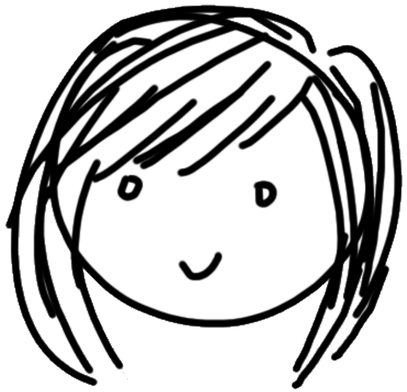

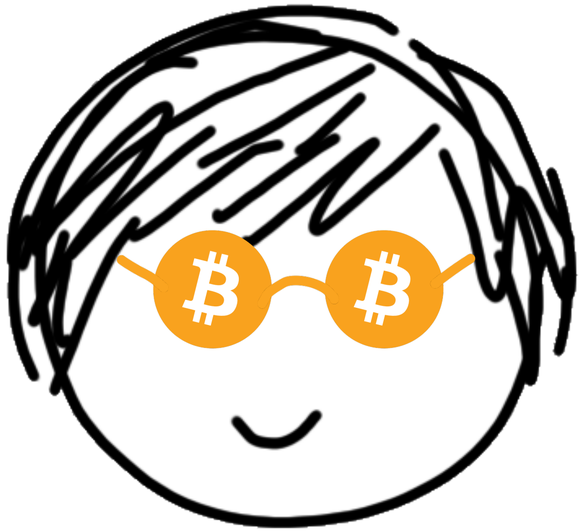

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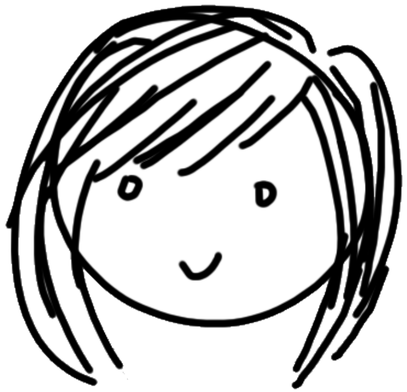

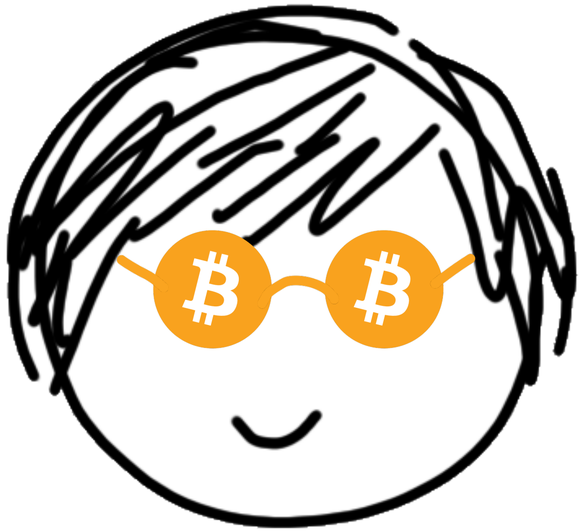

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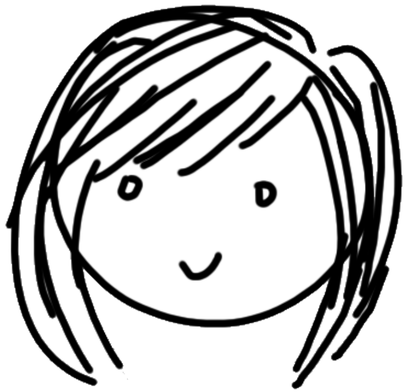

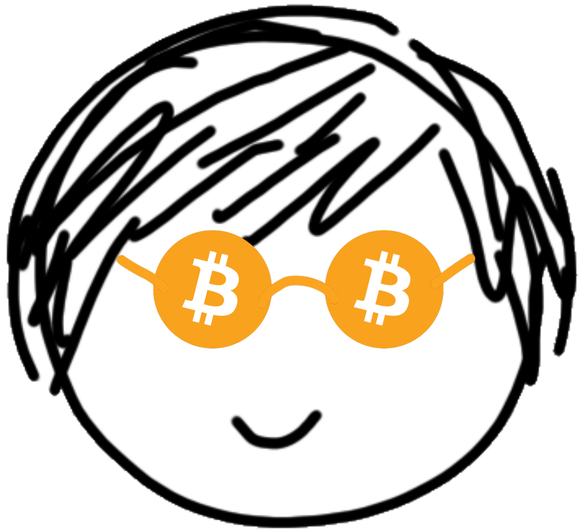

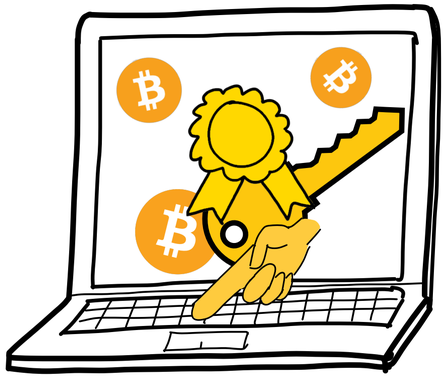

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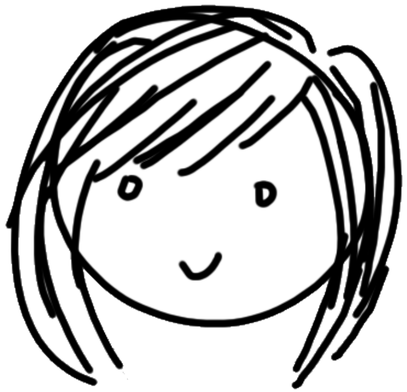

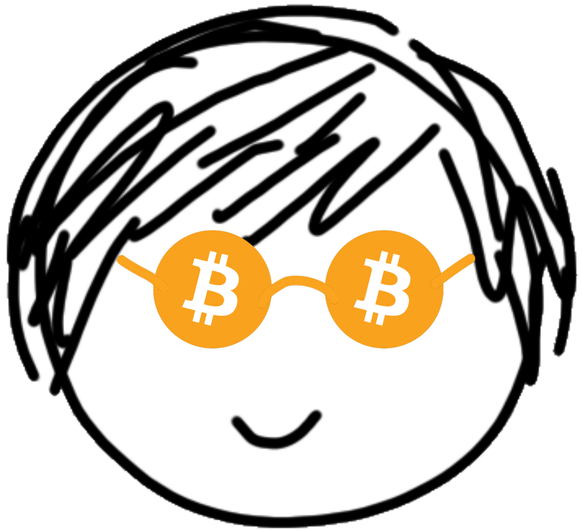

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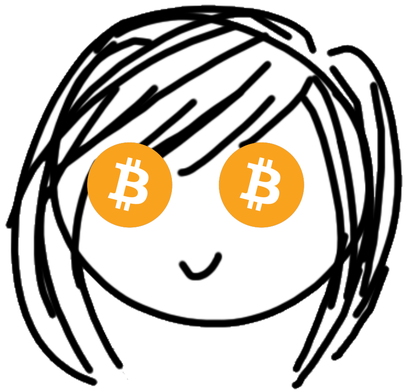

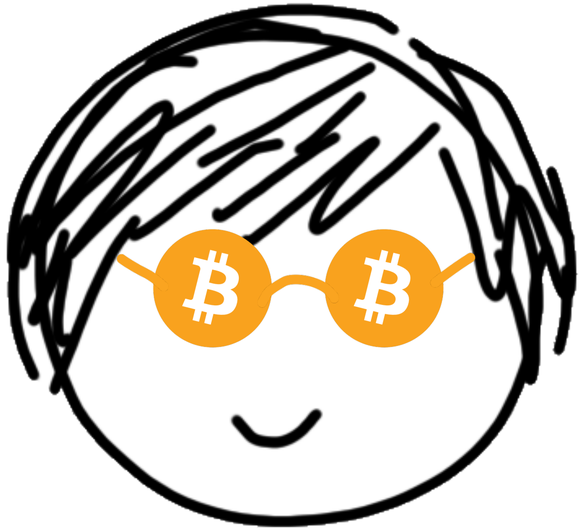

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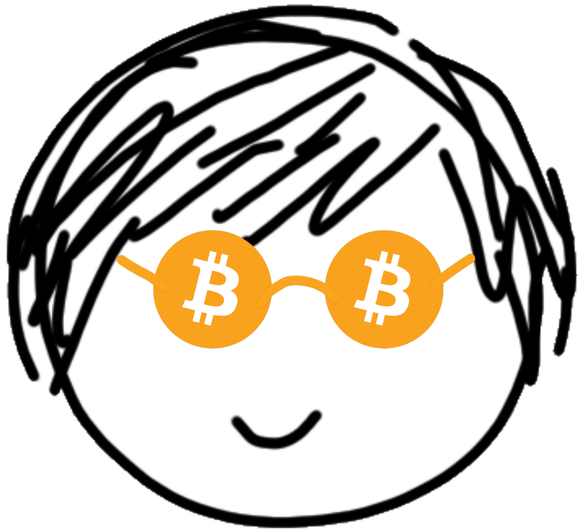

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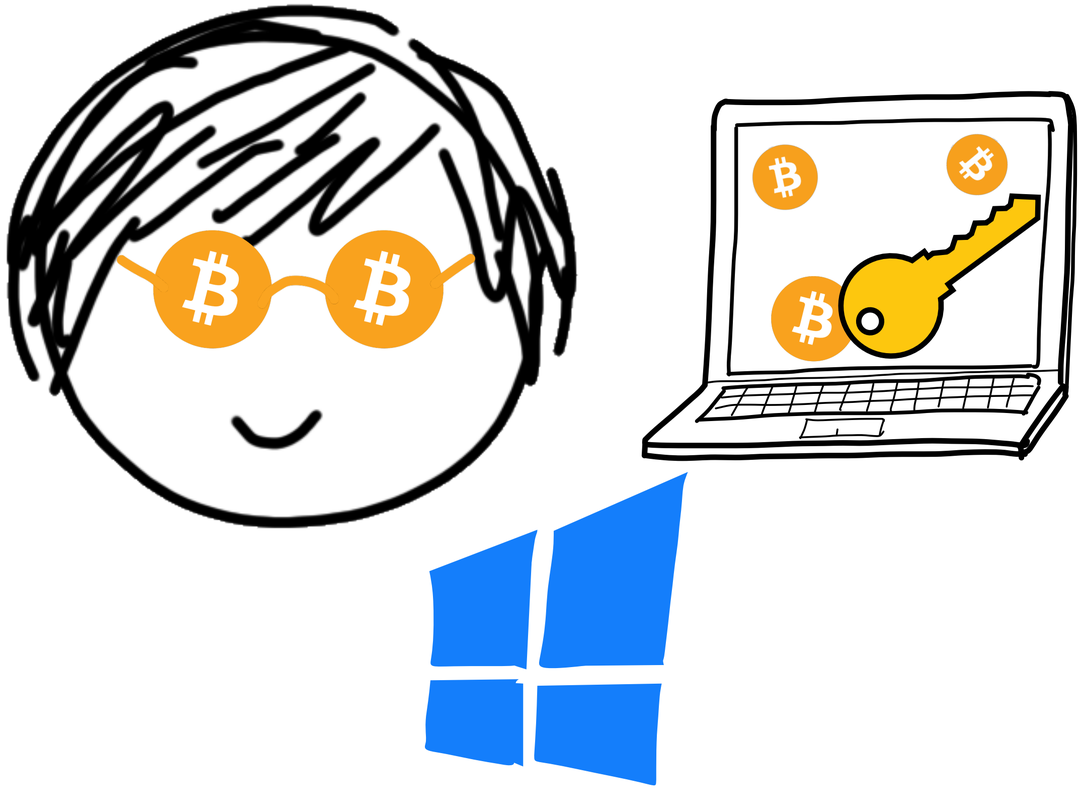

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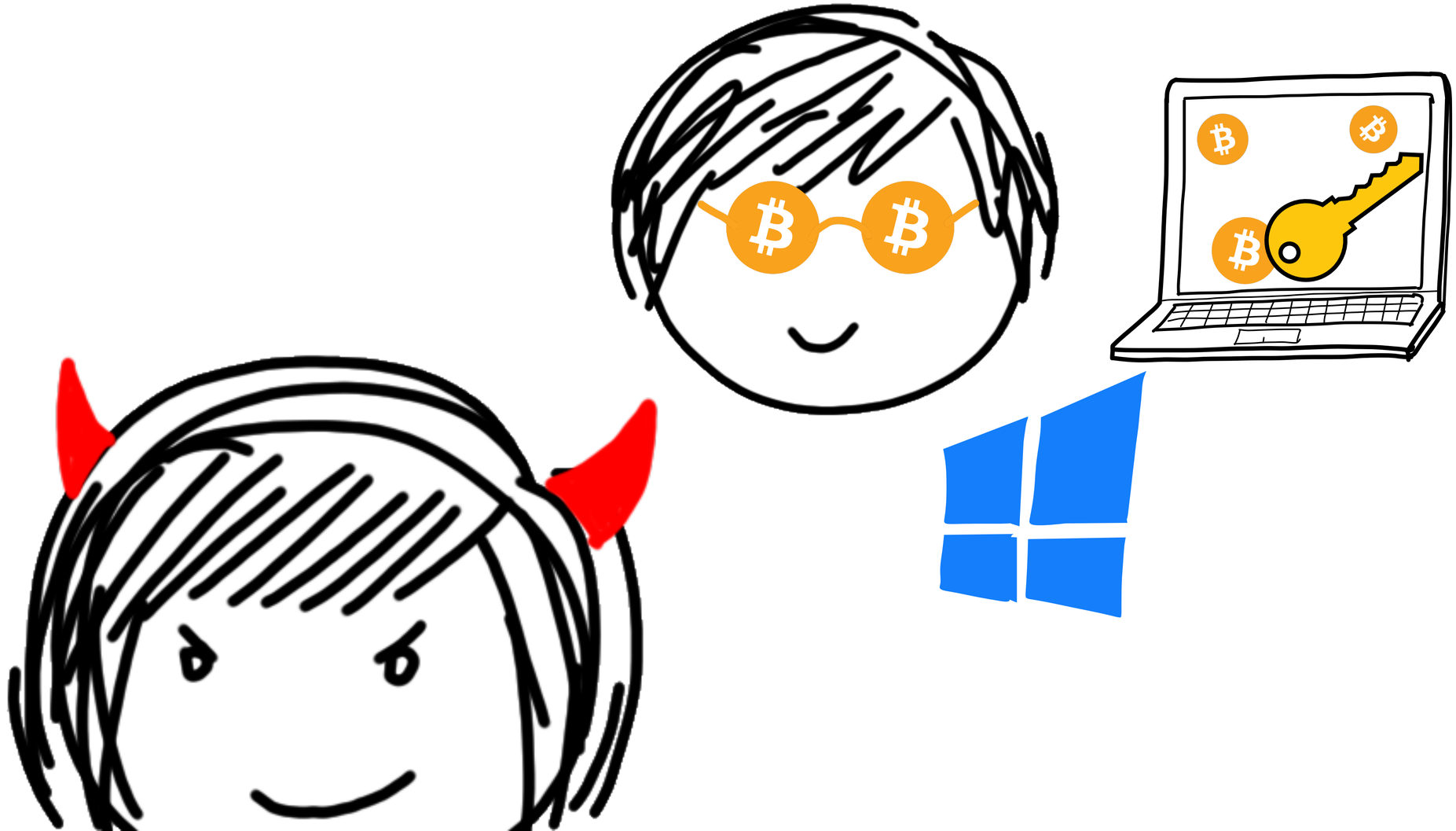

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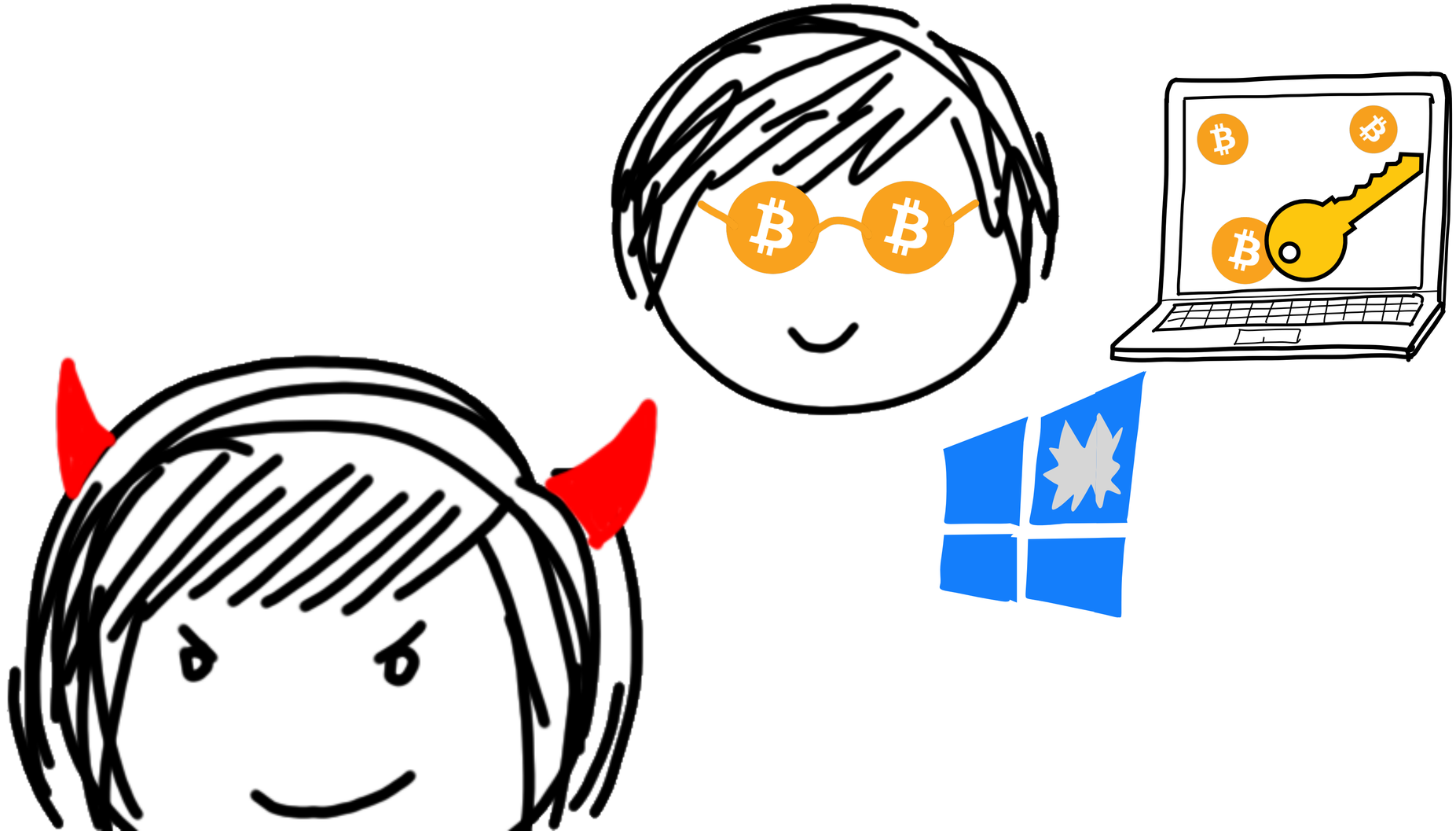

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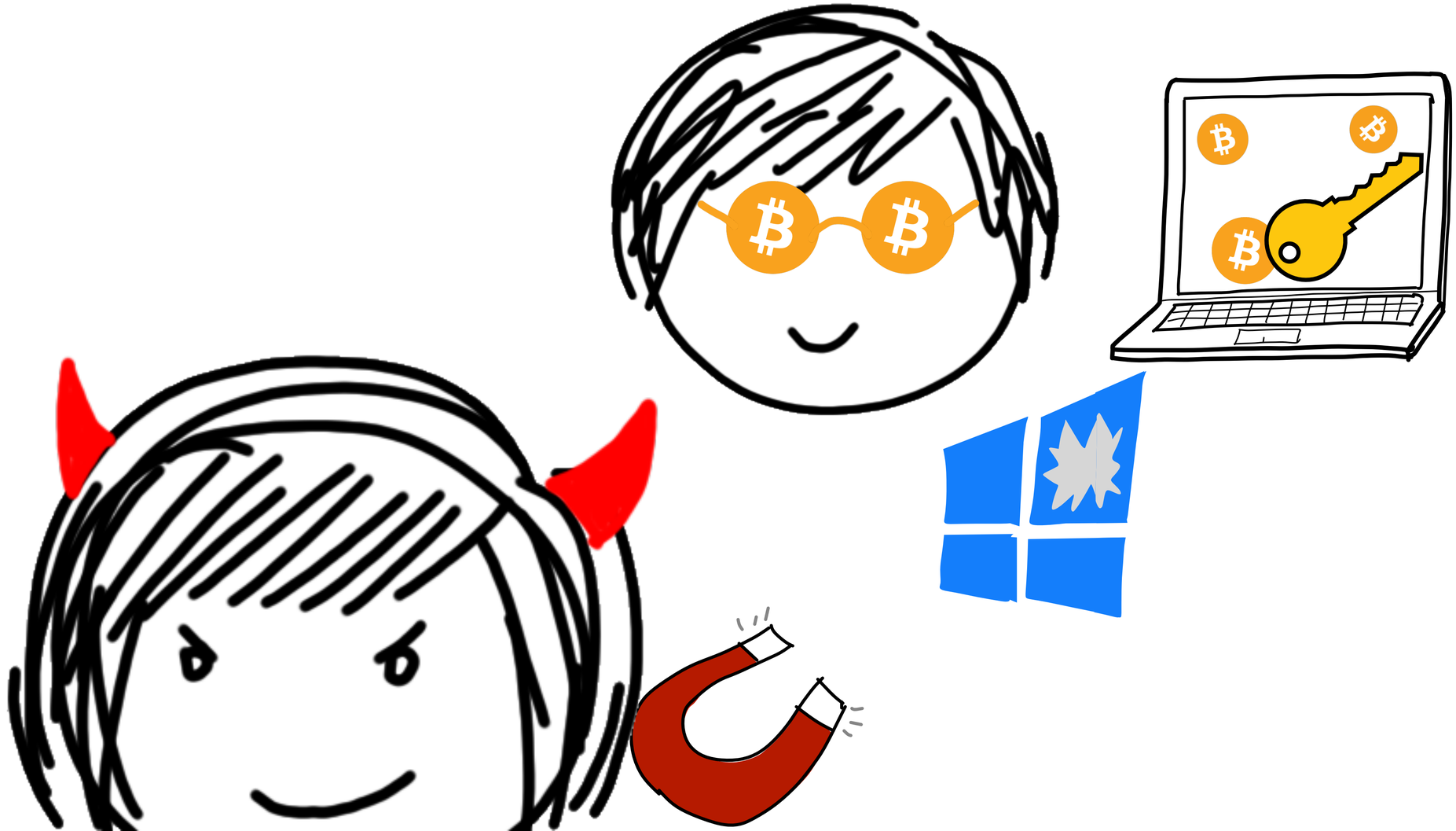

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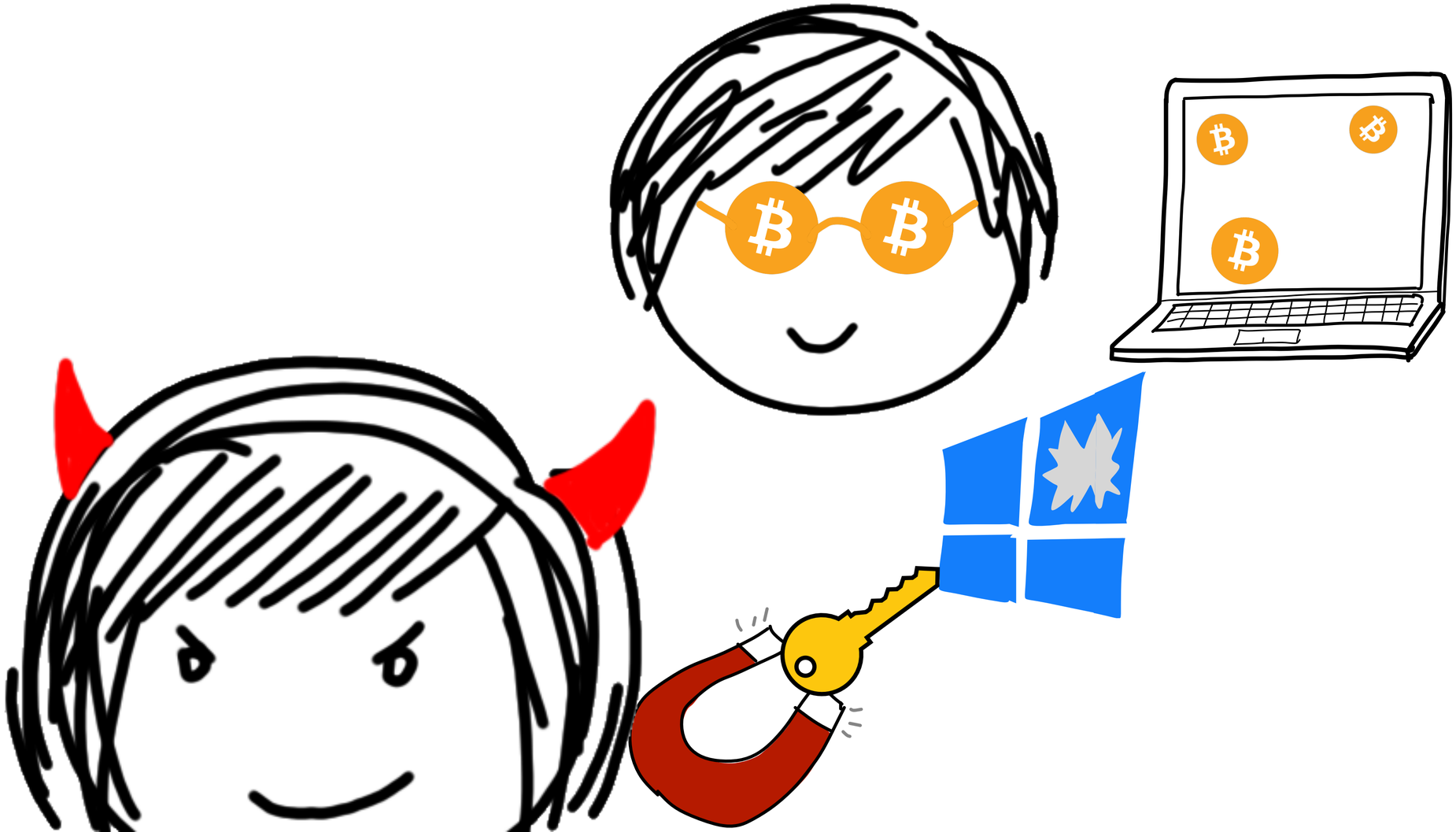

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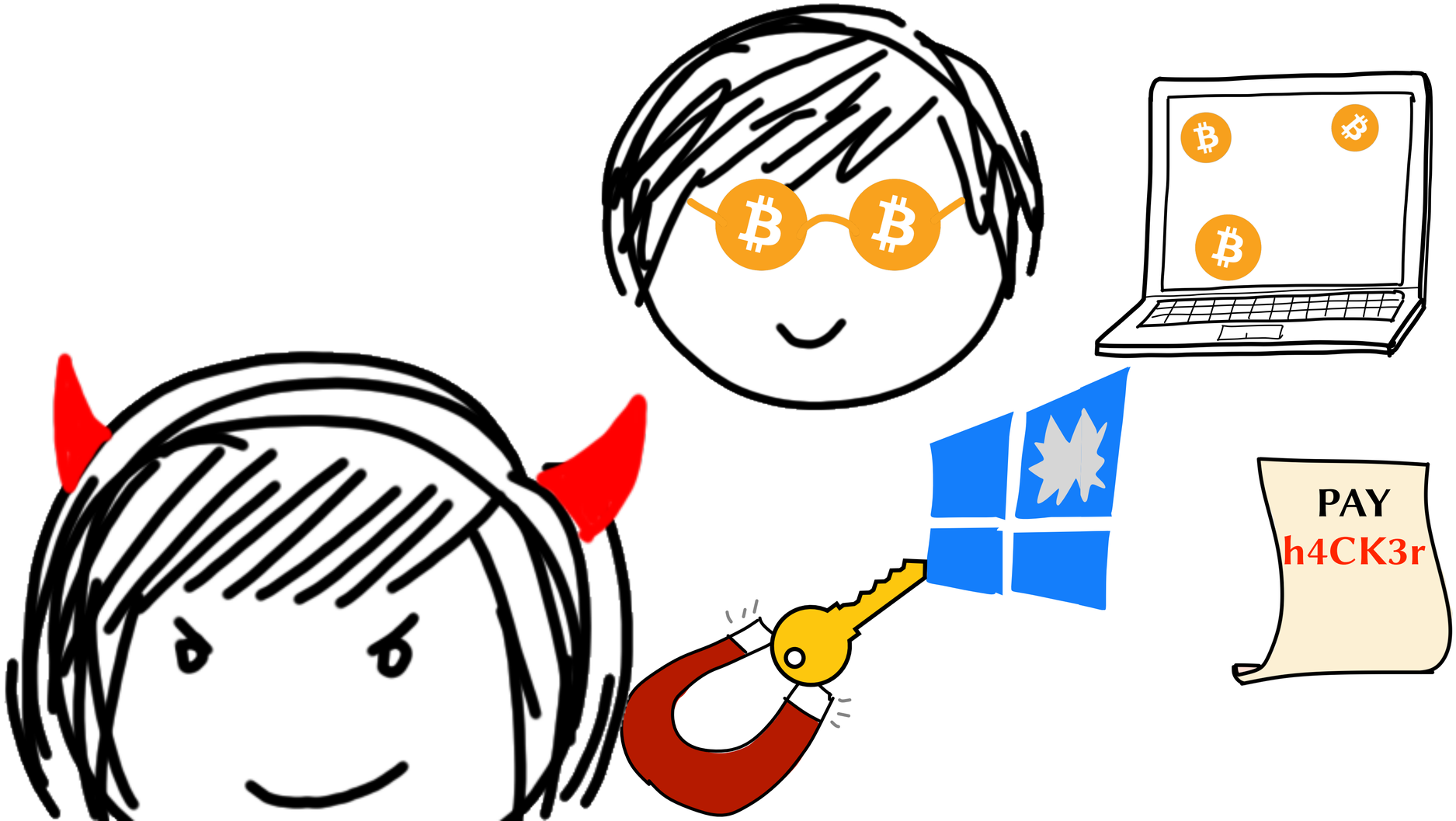

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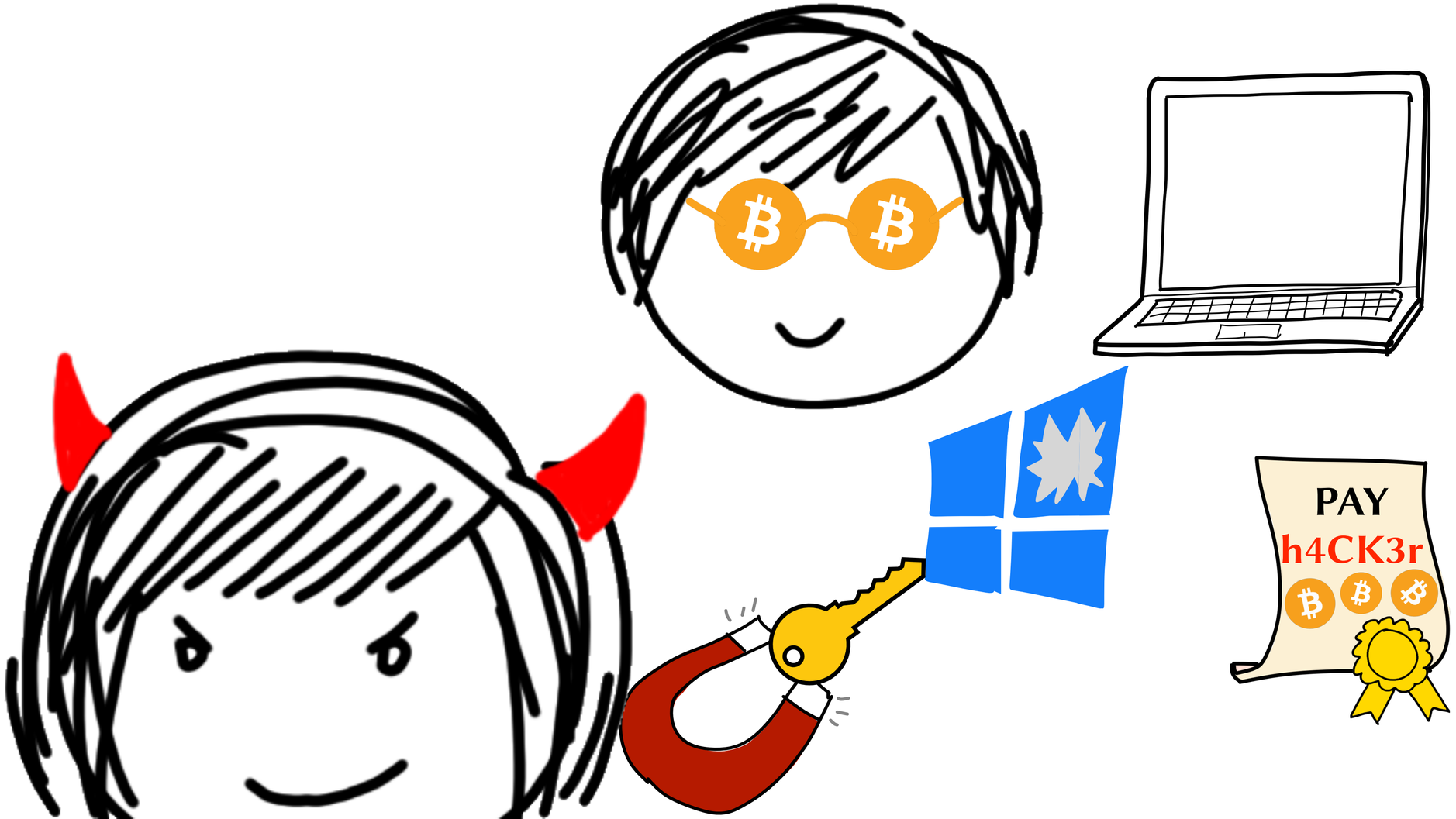

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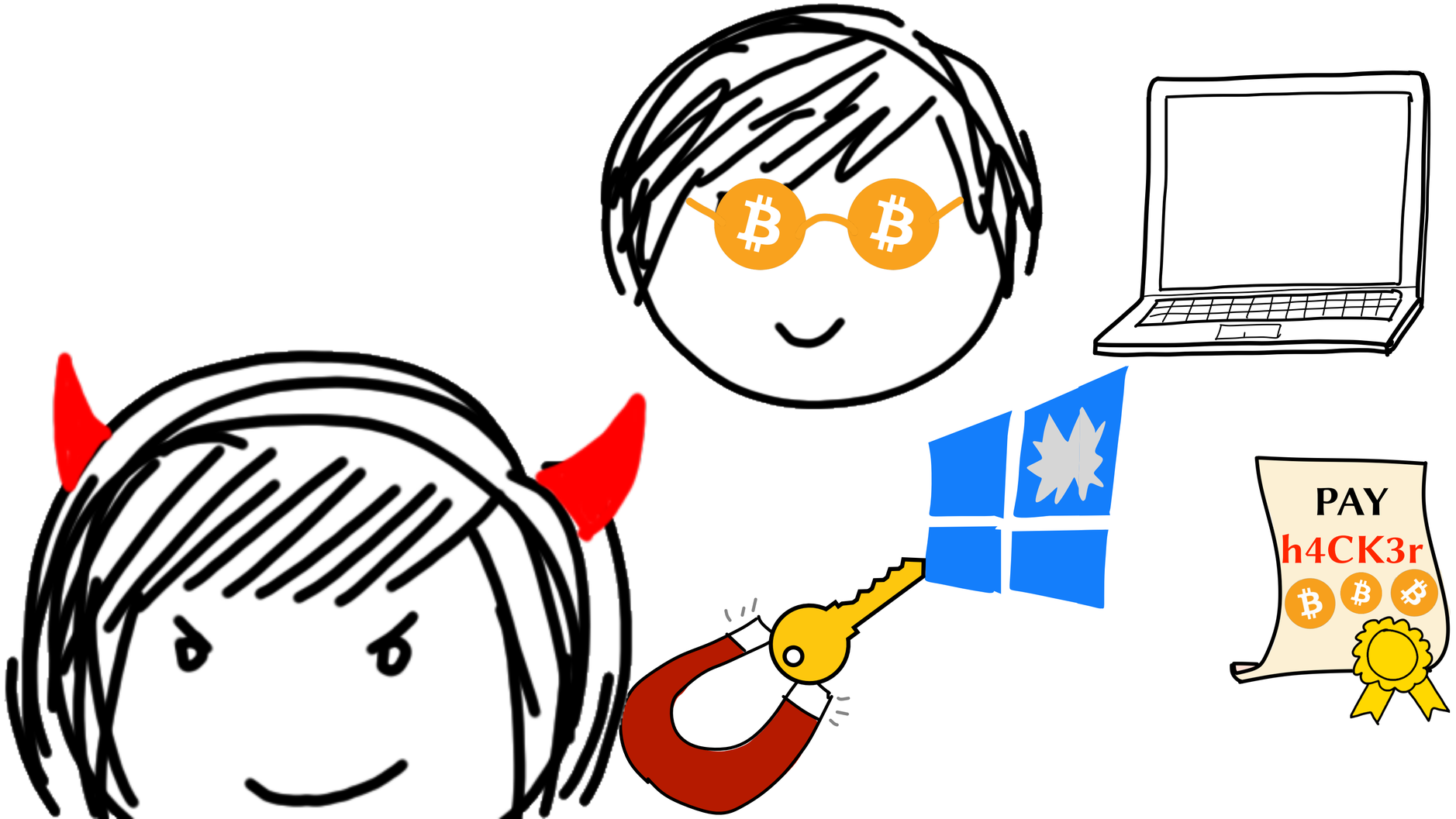

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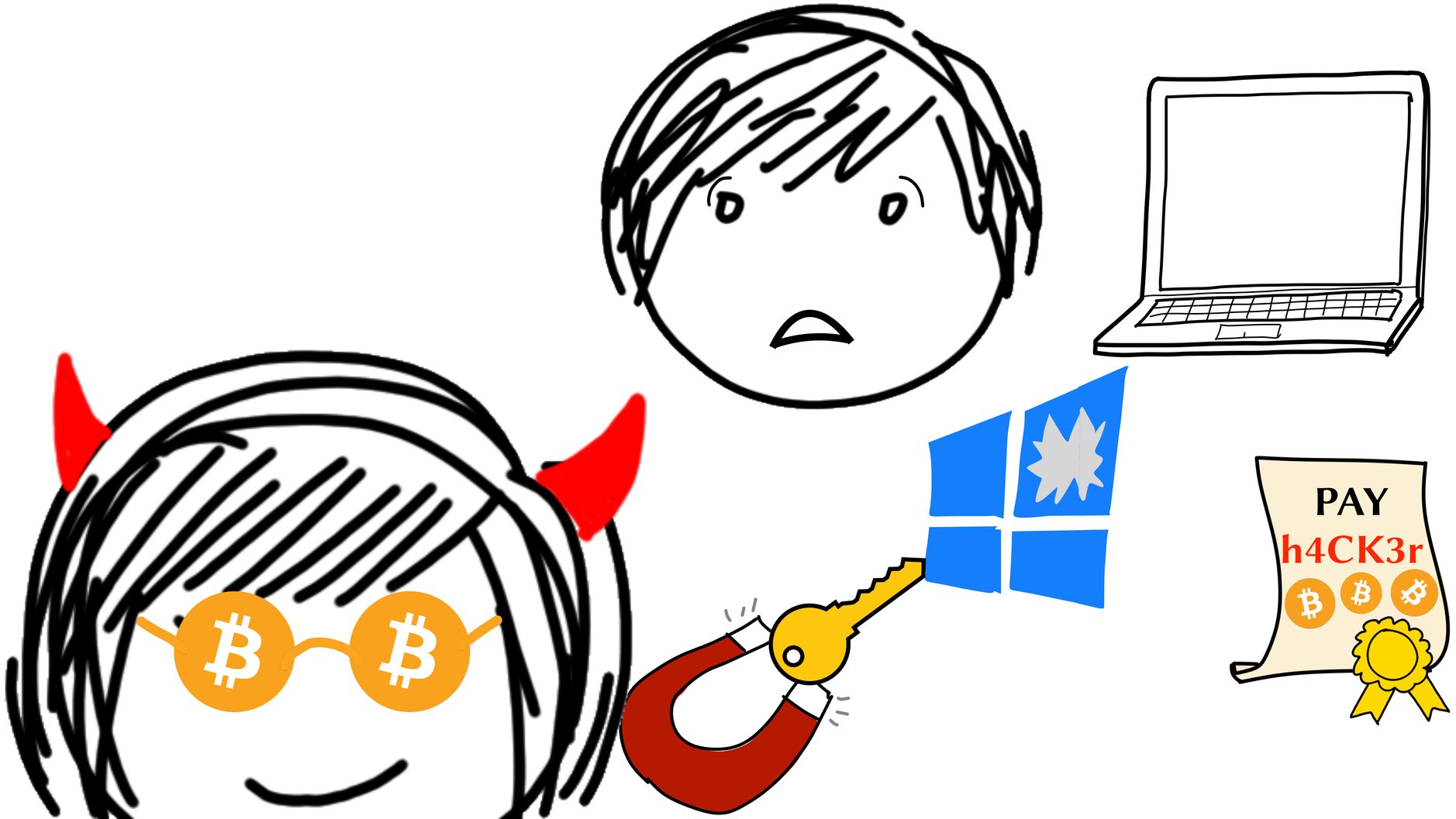

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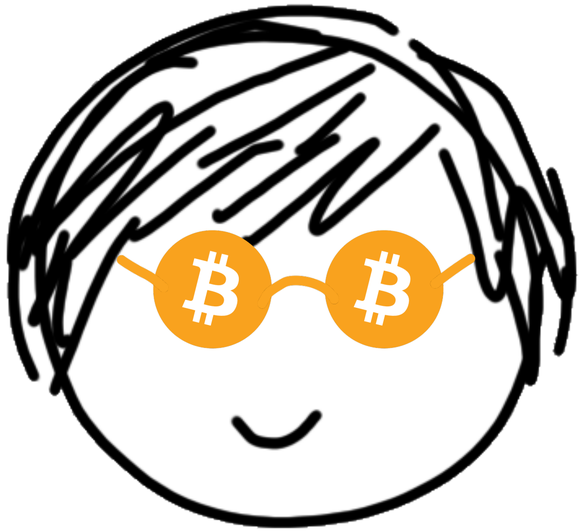

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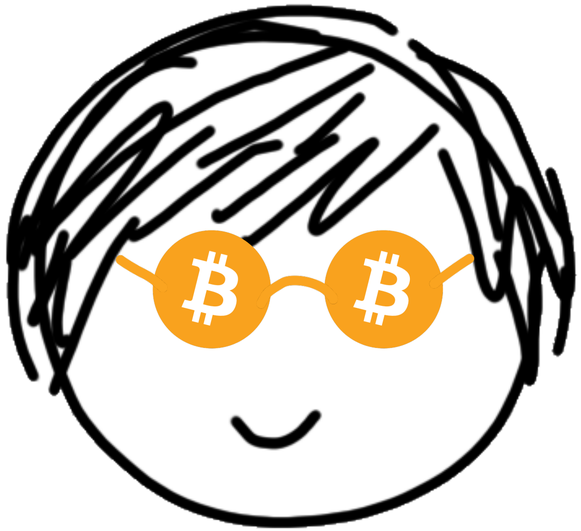

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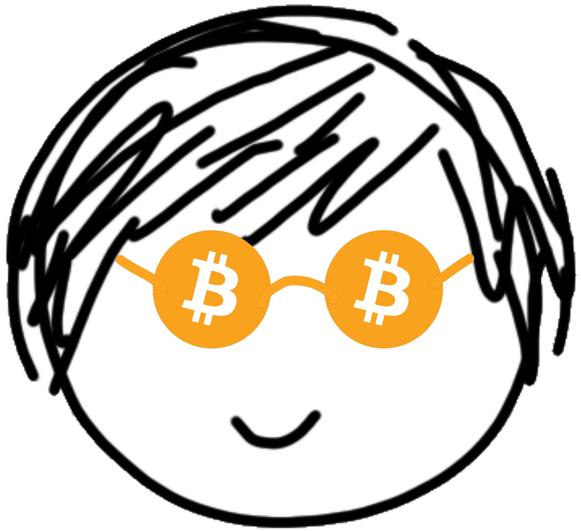

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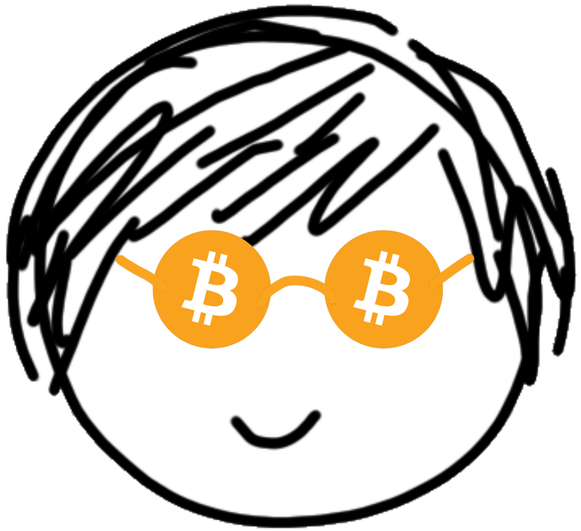

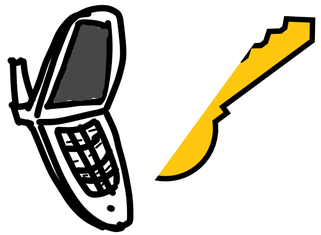

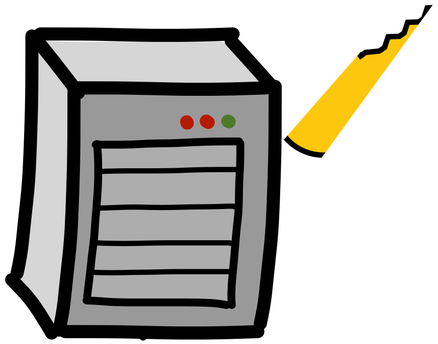

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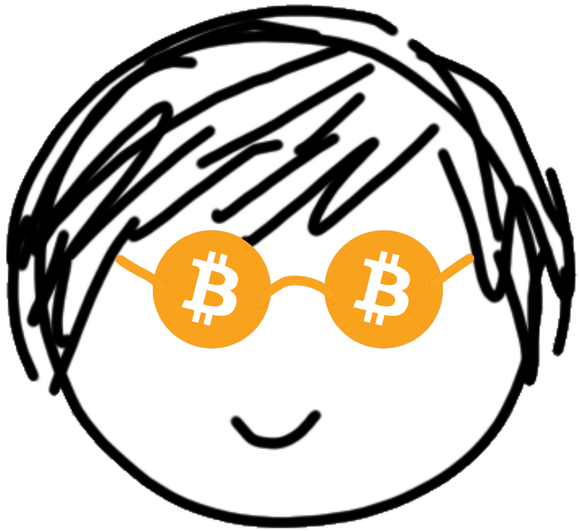

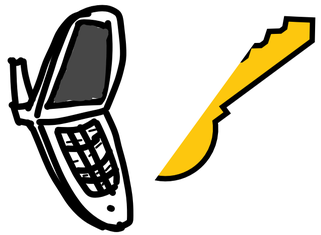

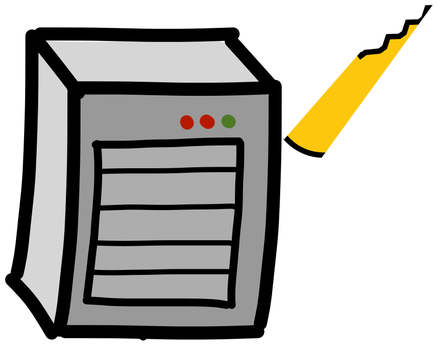

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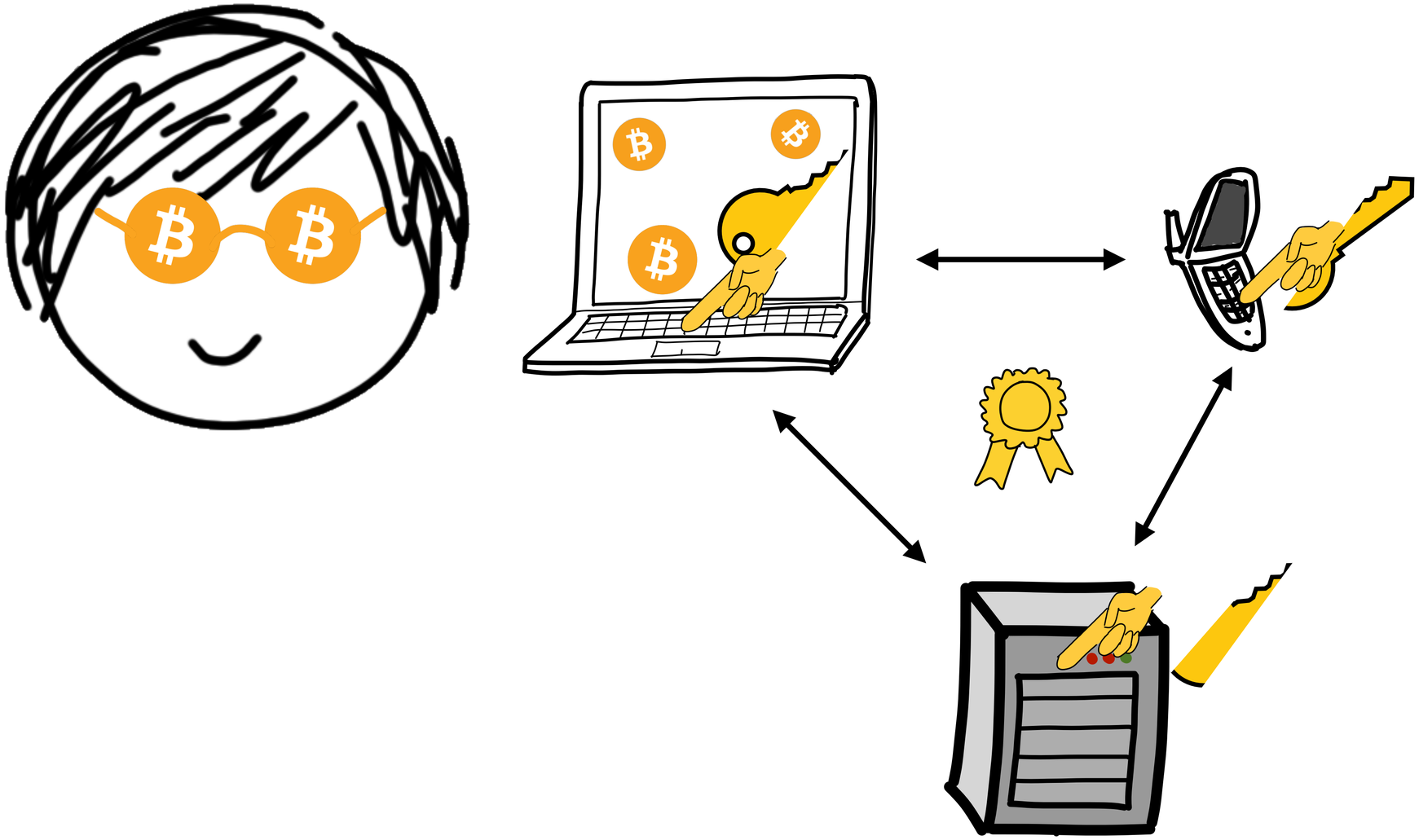

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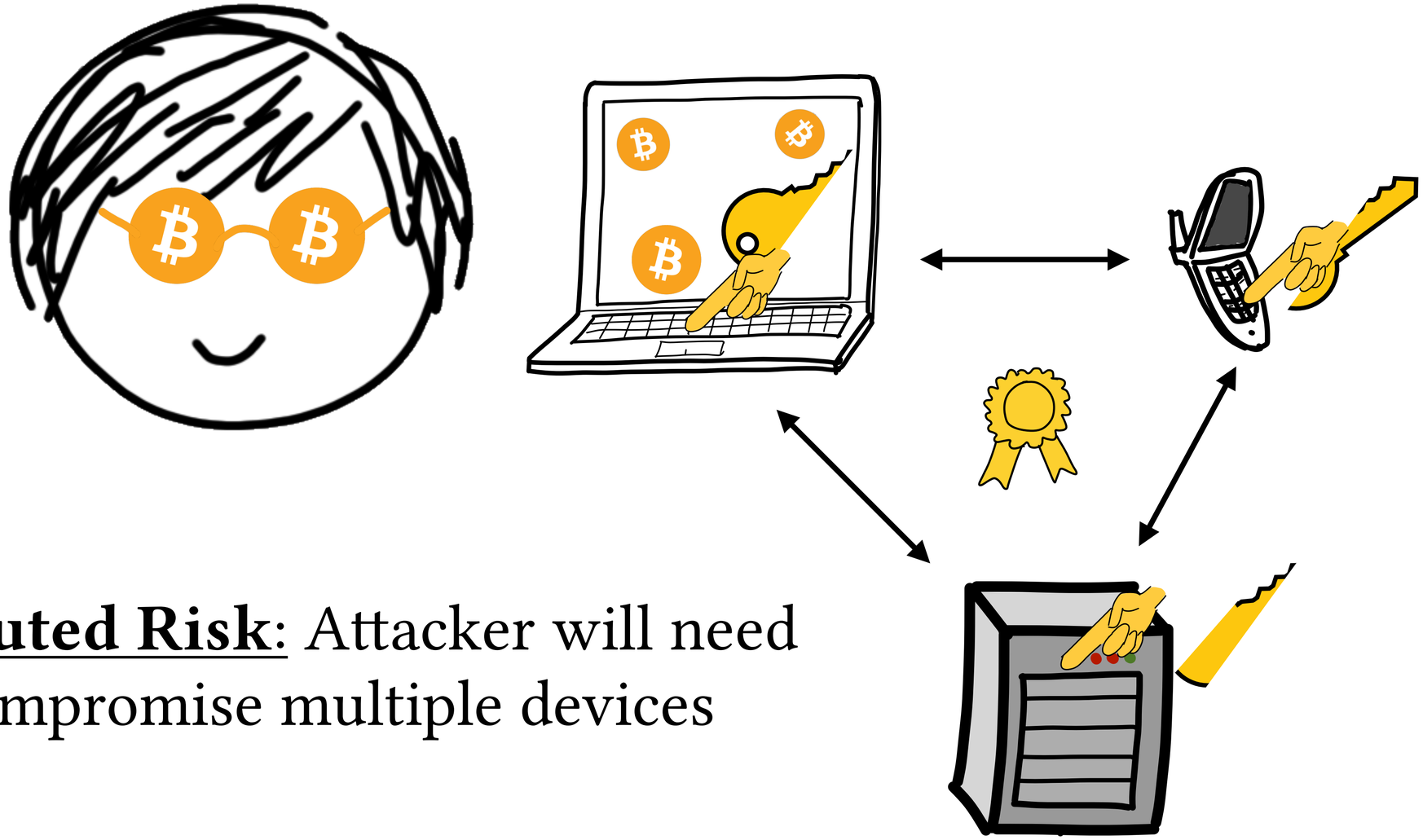

<u>Distributed Risk</u>: Attacker will need to compromise multiple devices

- **<u>Compatibility</u>**:
  Verifies w.r.t. original algorithm

- **<u>Corruption Resilience</u>**:
  Compromising some devices does not leak the signing key

- This talk: Signatures ⇔ Non-interactive Zero-knowledge

 $\Leftrightarrow \pi$

( , ) $\Leftrightarrow (x, w)$

Distributed Signing ⇔ Distributed Proving

How to Distribute Signing

- Any signing scheme can be distributed via general MPC
- "Practical" efficiency usually requires more fine-grained notions than just feasibility
- As one proxy, practical distributed signing protocols make **blackbox use** of non-linear components of the signing algorithm:
  - Integer arithmetic in $\mathbb{Z}_q$ or $\mathbb{Z}_N^*$
  - Elliptic curve group operations
  - Hash functions

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RSA, Schnorr/EdDSA, ECDSA, BLS, BBS+, custom constructions using lattices, isogenies, etc.

What about Purely Hash Based?

- [Ozdemir Boneh 22]: distributed version of Fractal
  [Cui Zhang Chen Liu Yu 21]: distributed MPC-in-the-head
  Proof size, verifier time linear in #provers

- [Khaburzaniya Chalkias Lewi Malvai 21]: aggregate Lamport signatures with STARKs
  Prove statements about circuit representation of hash function

- [Dziembowski Faust Lizurej 23]: "individual cryptography"
  Hash-based proofs that are designed to be hard to distribute

- [Nielsen Hall-Andersen 23]: Incrementally Verifiable Computation must make non-blackbox use of hash function

This Work: Limitations

- For some hash based NIZKs[1], there is an inherent barrier[2] to designing practical protocols[3] to distribute their computation.

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1. NIZKs that have straight-line extractors in the Random-Oracle Model
2. Attack that completely recovers the witness by corrupting all-but-one distributed provers
3. Protocol that is blackbox in the same hash function (i.e. Random Oracle) as the NIZK

Implications for distributing…

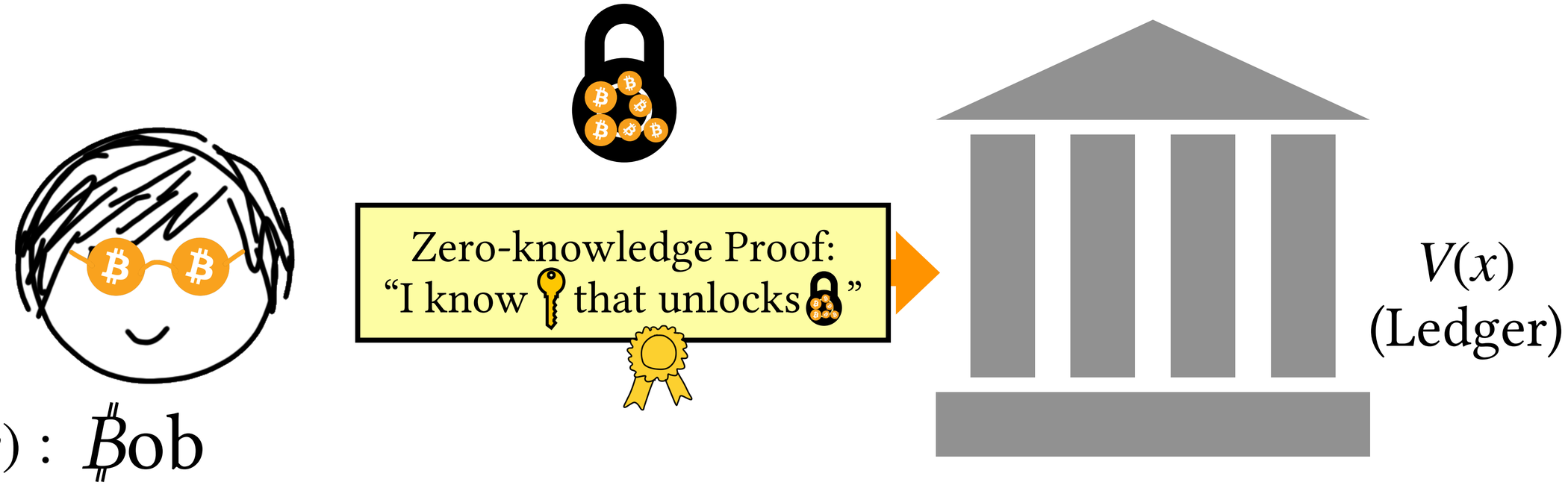
- Signing for standard schemes based on MPC-in-the-head
- NIZKs/signatures obtained by compiling Sigma protocols via:
  - Pass' or Fischlin's transformations (tight/concurrent security)
  - Unruh's transformation (post-quantum)
- PCPs/IOPs compiled via hash functions

Zero-knowledge Proofs

- Very powerful cryptographic primitive, introduced by [Goldwasser Micali Rackoff 85]
- Intuition: Prover convinces a Verifier of a statement, without revealing its secret trapdoor.
- In this talk, we only look at:
  - Non-interactive proofs (NIZK)
  - Proofs of Knowledge (PoK)

Zero-knowledge Proofs

- ZK is intuitive: No information about the key should be leaked by the proof
- But what does it mean to “know” something?
- “Proof of Knowledge” is formalized by an “extractor” Ext



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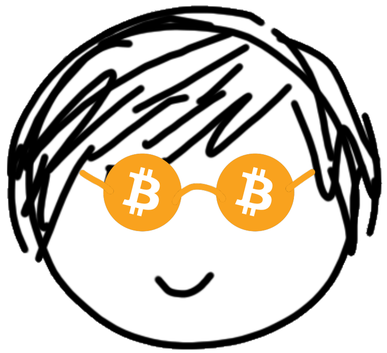

$P(x, w)$: Bob

Ext

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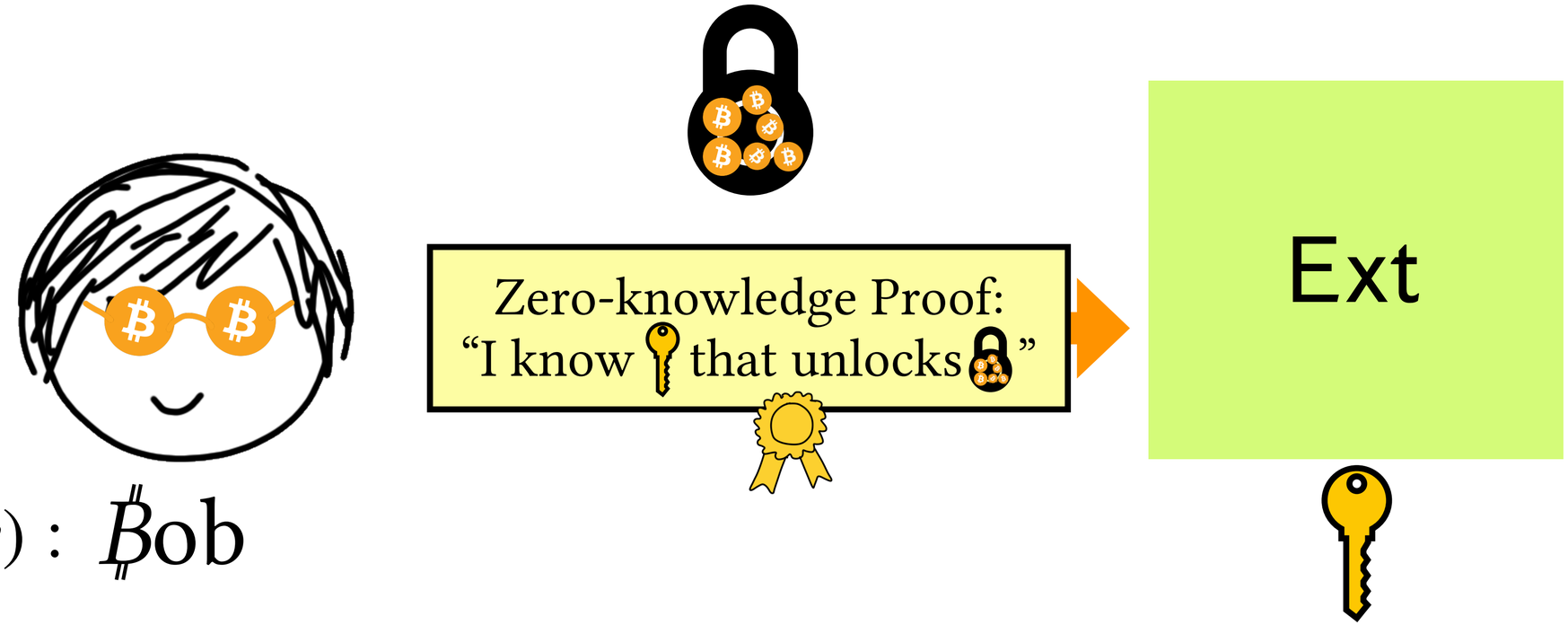

Why is Ext special?

- Clearly, Ext must not be an algorithm that just anybody can run
- Ext has carefully chosen special privileges:
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  - Still meaningful as a security claim
- “Straight-line” Extraction (SLE): no rewinding. Instead, use other trapdoor like CRS, RO, etc.

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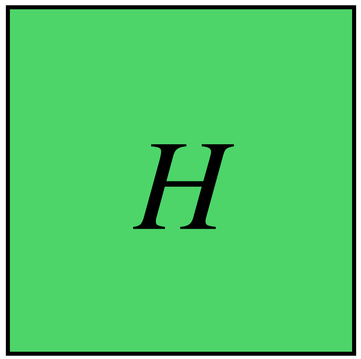
Bad for:

- Quantum
- Concurrency
- Tightness

Random Oracle Model



$$H : \{0,1\}^* \mapsto \{0,1\}^{\ell}$$

Random Oracles as Ext Privilege

H

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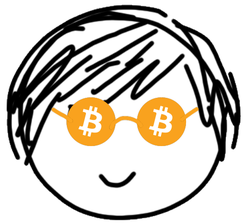

$P(x, w)$

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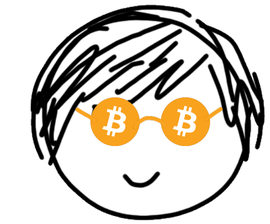

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Q_1	Q_2	•	•	Q_i	•	•	•	•	•	Q_n

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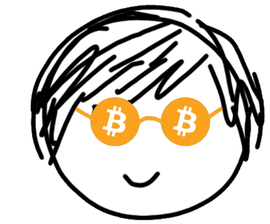

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$V(x)$

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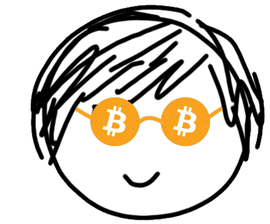

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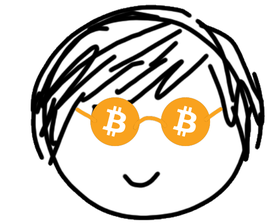

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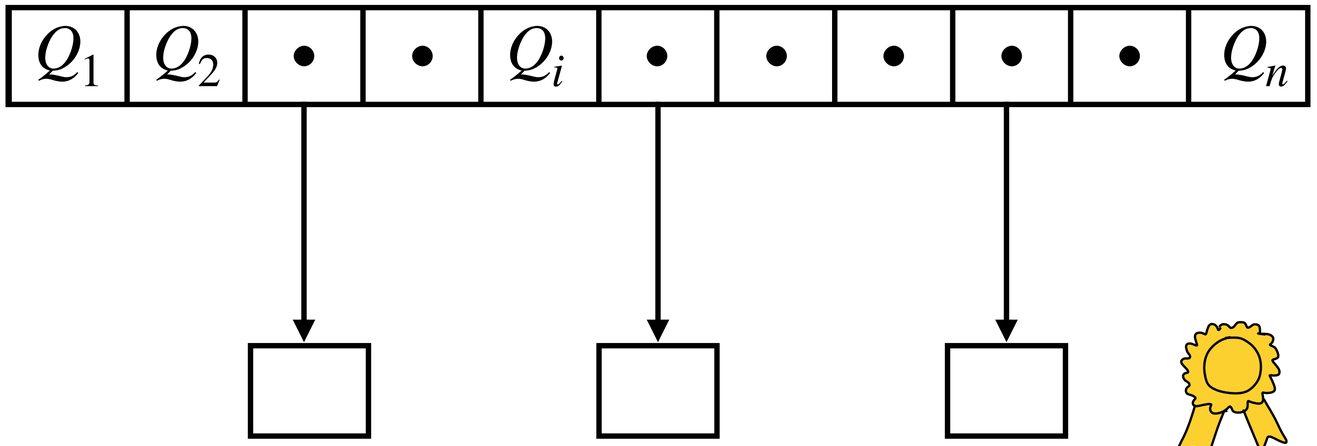

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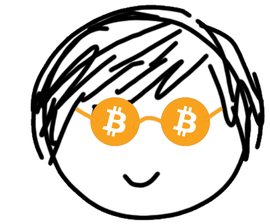

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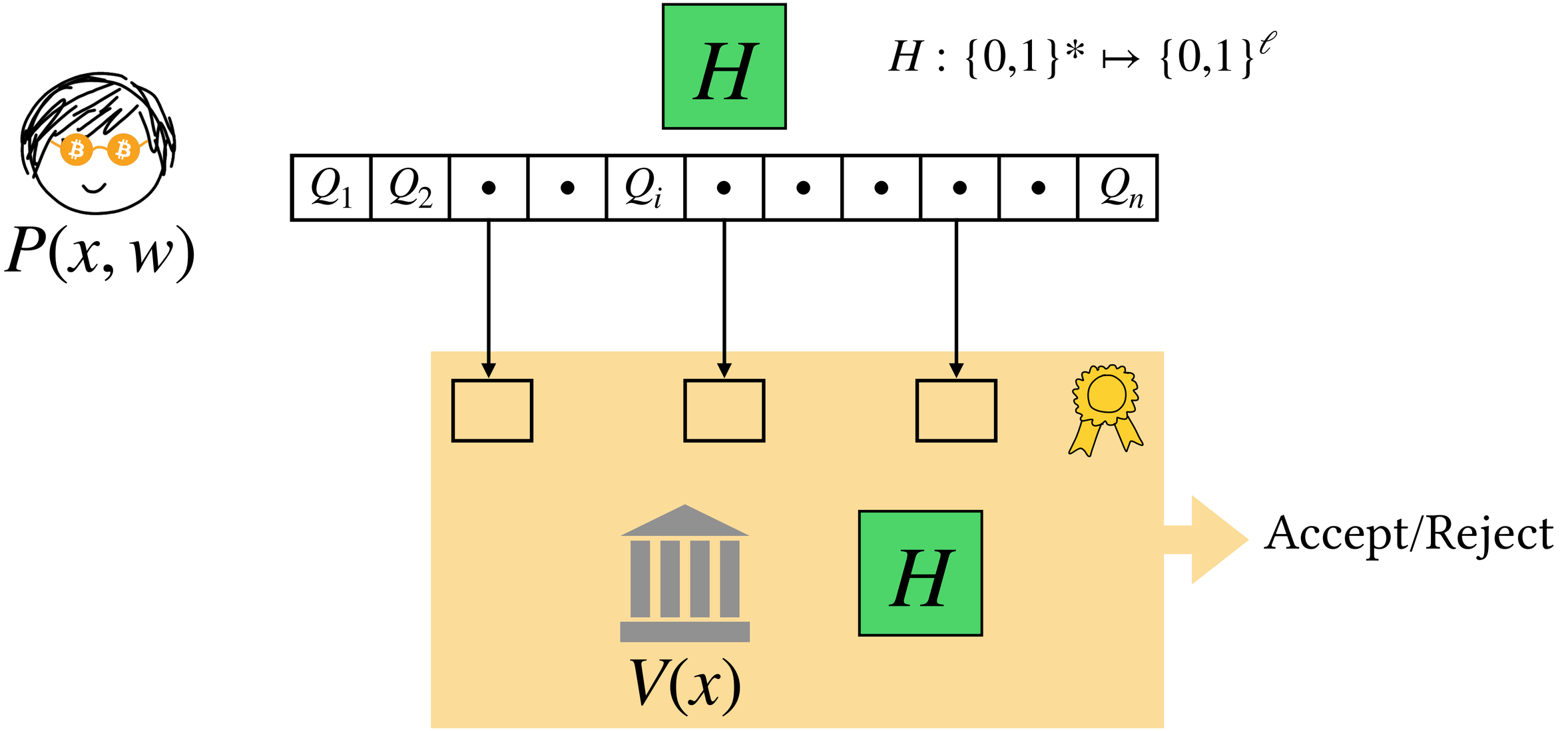

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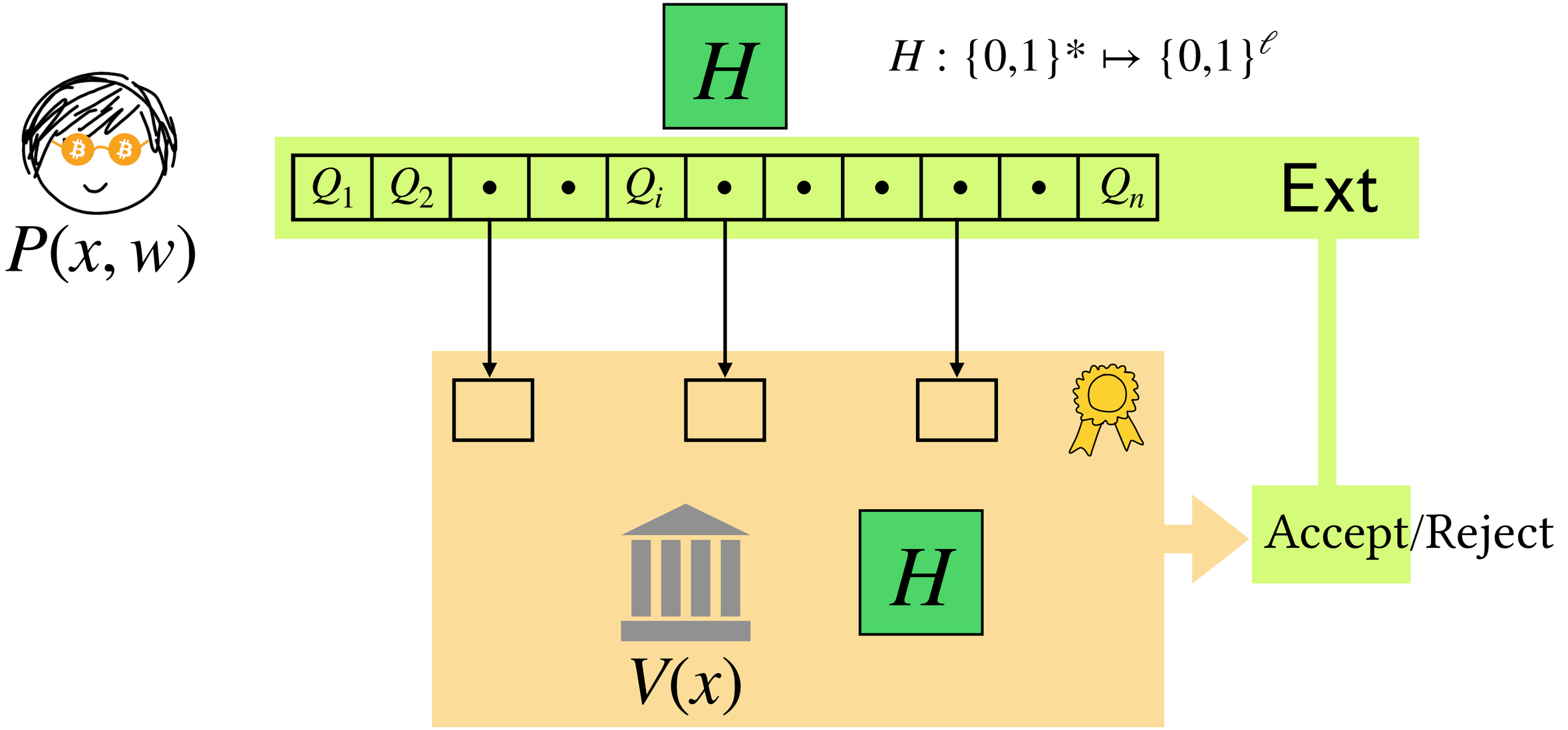

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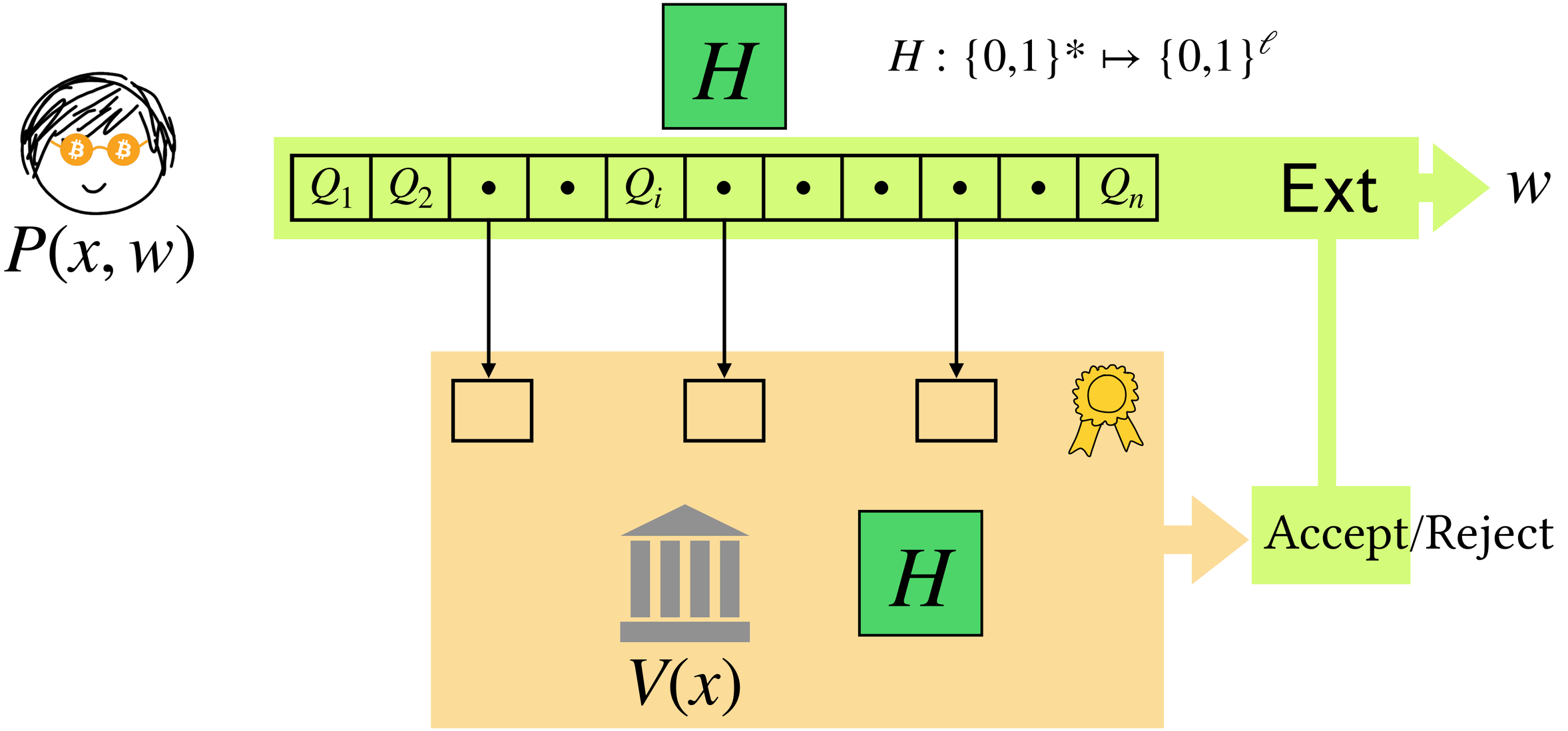

Random Oracles as Ext Privilege

- Why is it a meaningful trapdoor?
  - Hash functions are complex and highly unstructured
  - Bob must "query" each Q_i to H to obtain $H(Q_i)$
  - Ext gets $\{Q_i\}$ without rewinding
- Practical usage:
  - No "trusted setup", each query is very cheap
  - Many NIZKs **happen to achieve** SLE in the ROM

Distributing NIZKs in the ROM

- Multiparty protocols to securely compute RO-based NIZKs should ideally make blackbox use of H
  - Conceptually: H should not have a circuit description
  - Practically: hash functions have large circuits
- We call them "Oracle Respecting Distributed" (ORD) protocols

Trivial Oracle Respecting Distribution

$$\pi \leftarrow P(x, w) \qquad V(x, \pi) = 1$$

Consider languages where (x, w) can be "secret shared":

$x_0 + x_1 + x_2 = x \qquad w_0 + w_1 + w_2 = w$ (think DLog)

$(x_0, w_0), (x_1, w_1), (x_2, w_2) \in L \Leftrightarrow (x, w) \in L$

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$P^3(x, w)$:

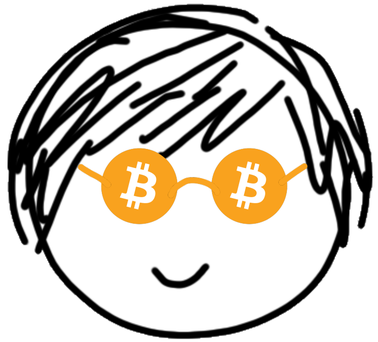
$w_0, w_1, w_2 \leftarrow \mathsf{Share}(w)$

Output $\{\pi_i = P(x_i, w_i)\}_{i \in [3]}$

$V^3(x, \pi_0, \pi_1, \pi_2)$:

$V(x_0, \pi_0) \wedge V(x_1, \pi_1)$
$\wedge V(x_2, \pi_2)$

Trivial Oracle Respecting Distribution



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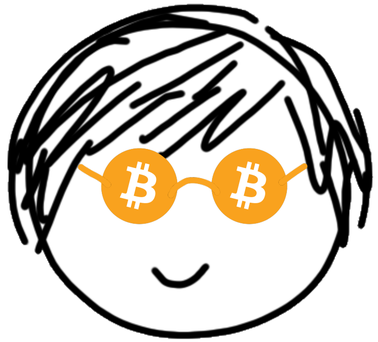
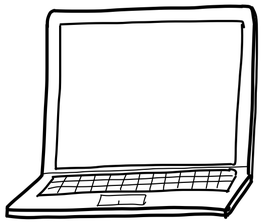
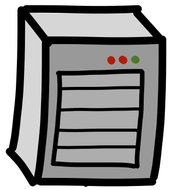


$P^3(x, w)$:



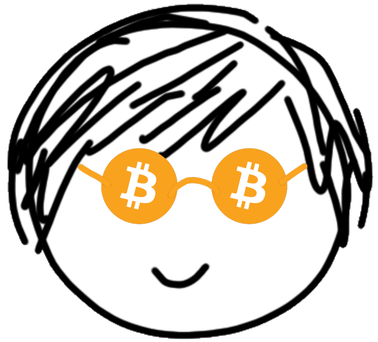
w_0



w_1

w_2

Trivial Oracle Respecting Distribution



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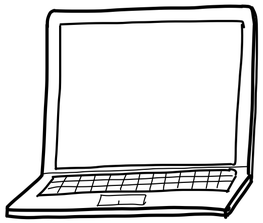
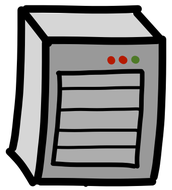
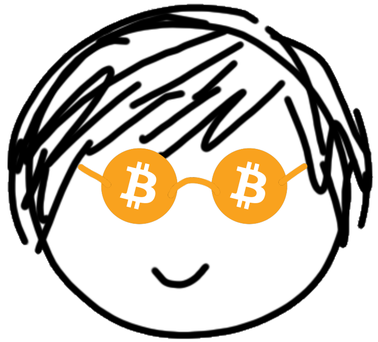




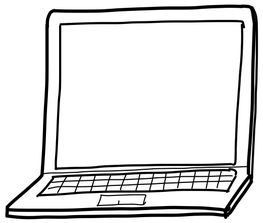
w_0 w_1 w_2

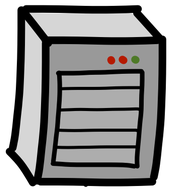
$\pi_0 \leftarrow P(w_0)$ $\pi_1 \leftarrow P(w_1)$ $\pi_2 \leftarrow P(w_2)$

Trivial Oracle Respecting Distribution



$P^3(x, w)$:



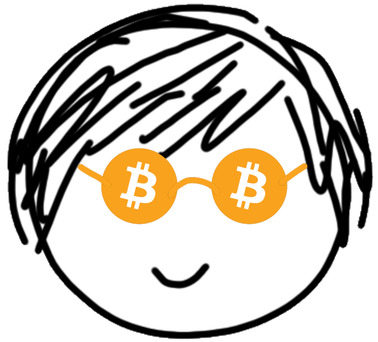


w_0 w_1 w_2

$\pi_0 \leftarrow P(w_0)$ $\pi_1 \leftarrow P(w_1)$ $\pi_2 \leftarrow P(w_2)$

Output (π_0, π_1, π_2)

Trivial Oracle Respecting Distribution



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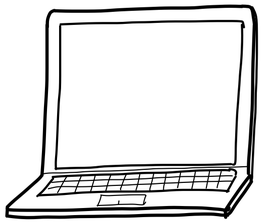
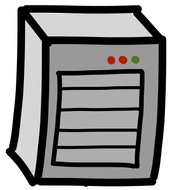


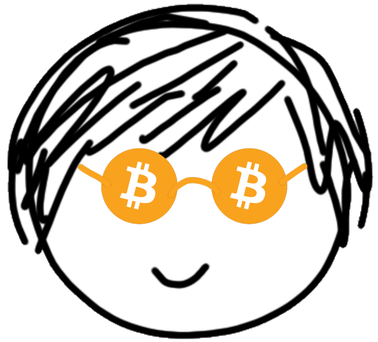


H w_0 $\quad$ H w_1 $\quad$ H w_2

$\pi_0 \leftarrow P(w_0)$ $\quad$ $\pi_1 \leftarrow P(w_1)$ $\quad$ $\pi_2 \leftarrow P(w_2)$

Output (π_0, π_1, π_2)

Trivial Oracle Respecting Distribution



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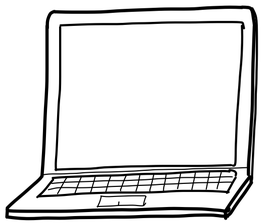
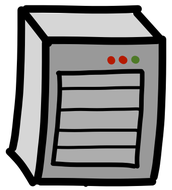




H w_0 $\quad$ H w_1 $\quad$ H w_2

$\pi_0 \leftarrow P(w_0)$ $\quad$ $\pi_1 \leftarrow P(w_1)$ $\quad$ $\pi_2 \leftarrow P(w_2)$

Additive secret sharing:
Resilience to two corruptions

Output (π_0, π_1, π_2)

Oracle Respecting Distribution: Notes

- Imagine if P^3 had to be distributed among *four* parties instead of three
- In general: P^* that outputs $n \times \pi$ can be distributed amongst n parties, **as long as V^* is aware of n**
- We show that for any NIZK that is SLE in the ROM, this is inherent in the $n - 1$ corruption setting

V "unaware" of n

- Consider a proof system (P^H, V^H) for some language
- Assumption: $n \in \mathsf{poly}(\kappa)$ is a strict upper bound on queries made by V to the random oracle H
  - Holds for most 'natural' schemes
- We will show: any $n + 1$-party protocol that ORD-computes P^H will leak the witness to n parties

Trimming Resilience

$P(x, w)$

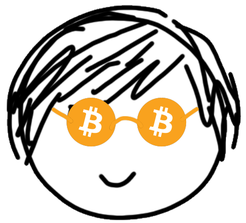
H

Q_1	Q_2	Q_3	Q_4	Q_5	Q_6	Q_7	Q_8	Q_9

π

Trimming Resilience

H



$P(x, w)$

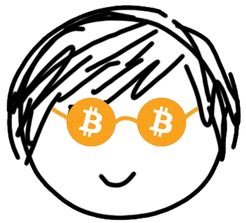
Q_1	Q_2	Q_3	Q_4	Q_5	Q_6	Q_7	Q_8	Q_9

π

$V(x)$

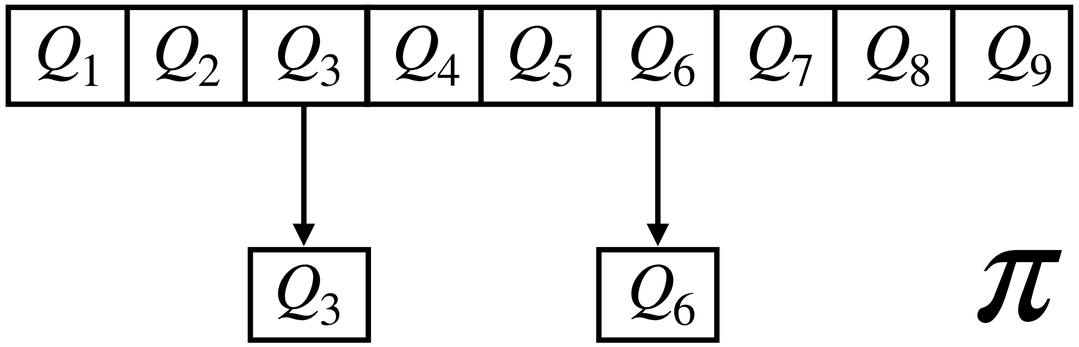
V checks at most $n = 2$ queries

Trimming Resilience



$P(x, w)$

H



π

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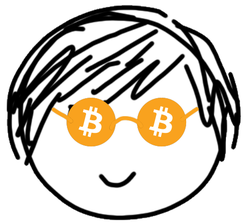
Trimming Resilience



$P(x, w)$

H

Q_1	Q_2	Q_3	Q_4	Q_5	Q_6	Q_7	Q_8	Q_9

Q_3 Q_6

π

$V(x)$

H

V checks at most $n = 2$ queries

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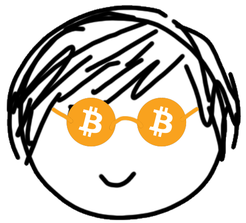

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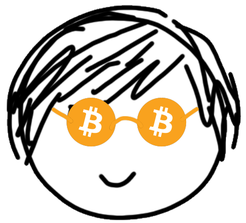


$P(x, w)$

H

At most two partitions will be touched by V

Q_1	Q_2	Q_3	Q_4	Q_5	Q_6	Q_7	Q_8	Q_9

$V(x)$

H

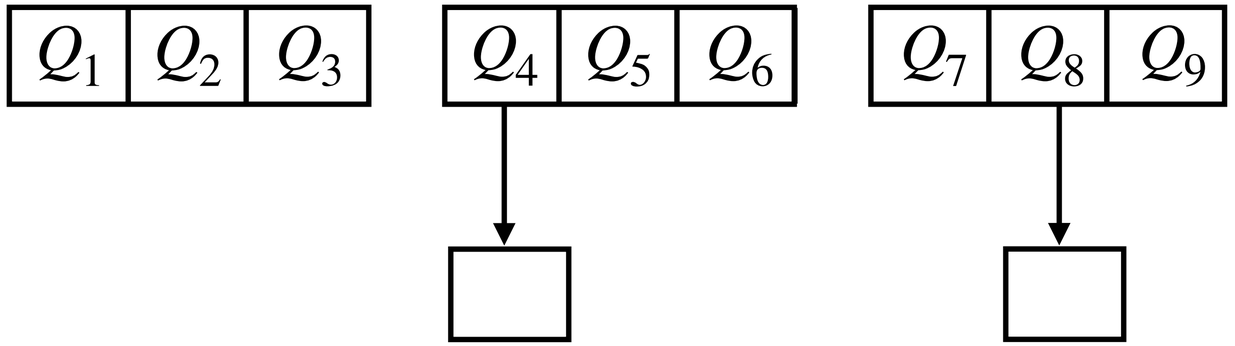
V checks at most $n = 2$ queries

Trimming Resilience

$P(x, w)$

H

At most two partitions will be touched by V

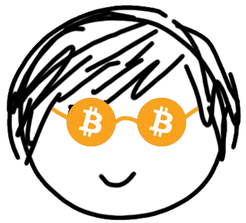


$V(x)$

H

V checks at most $n = 2$ queries

Trimming Resilience



$P(x, w)$

H

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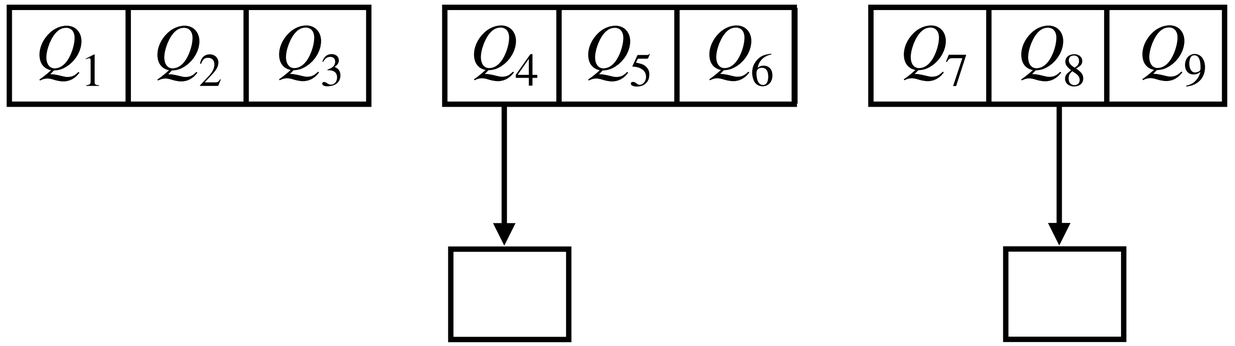


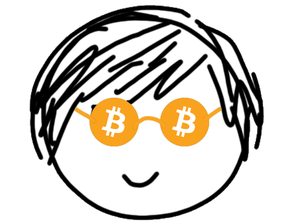
Randomly selected partition:
Pr[untouched by V] $\geq 1/3$

$V(x)$

H

V checks at most $n = 2$ queries

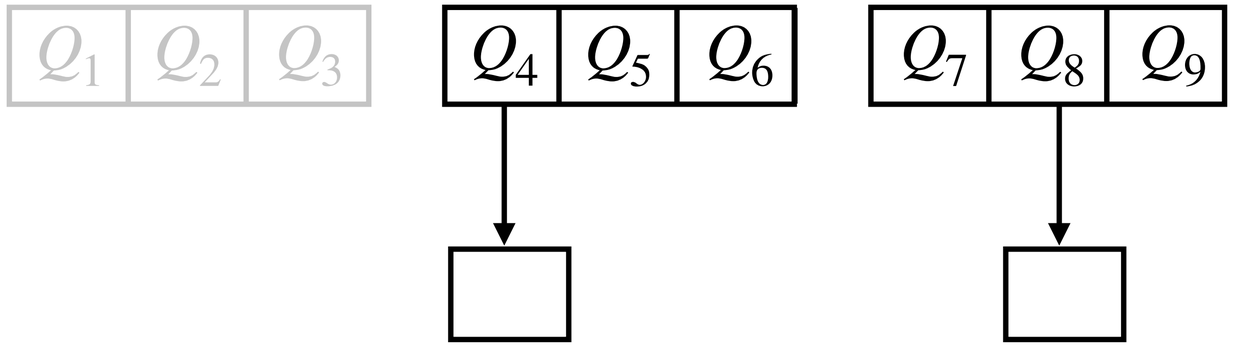
Trimming Resilience



$P(x, w)$

H

At most two partitions will be touched by V



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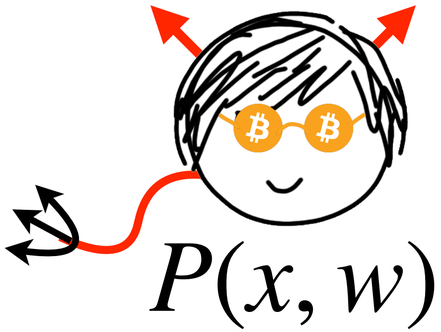
$V(x)$

H

V checks at most $n = 2$ queries

Trimming Resilience

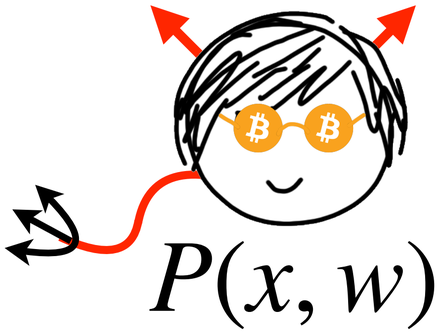


H

Q_1	Q_2	Q_3	Q_4	Q_5	Q_6	Q_7	Q_8	Q_9

V checks at most $n = 2$ queries

Trimming Resilience



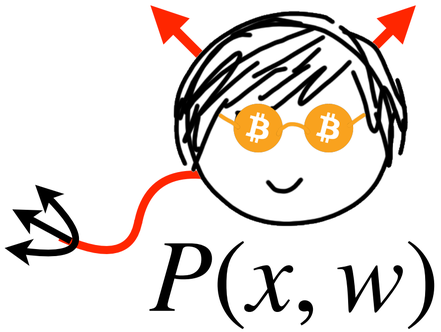
H^*

H

Q_1	Q_2	Q_3	Q_4	Q_5	Q_6	Q_7	Q_8	Q_9

V checks at most $n = 2$ queries

Trimming Resilience



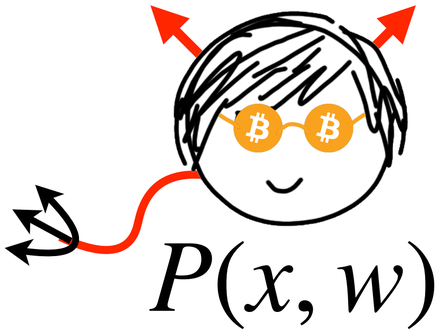
H

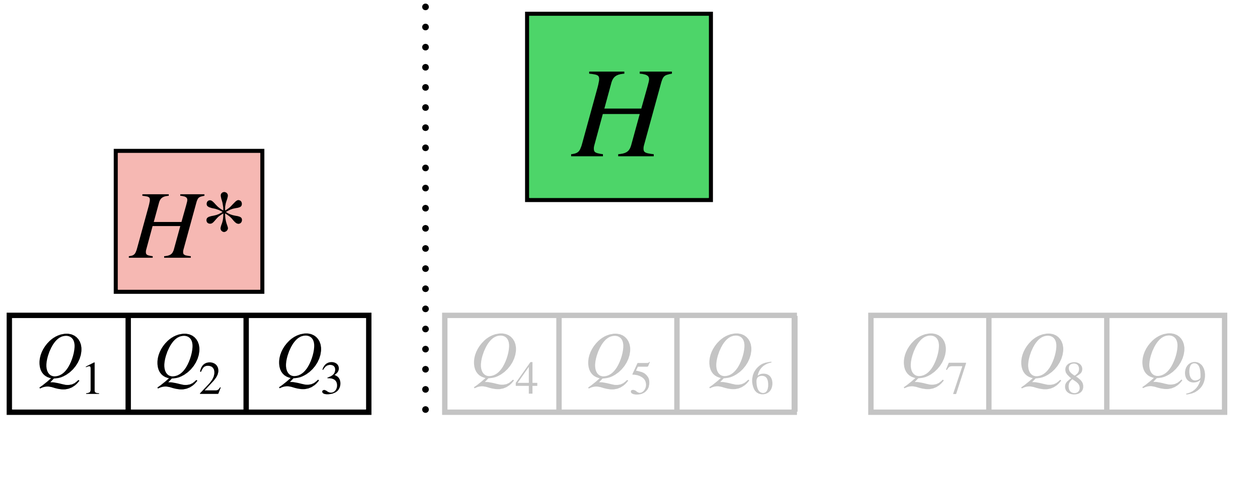
H^*

Q_1 Q_2 Q_3 Q_4 Q_5 Q_6 Q_7 Q_8 Q_9

V checks at most $n = 2$ queries

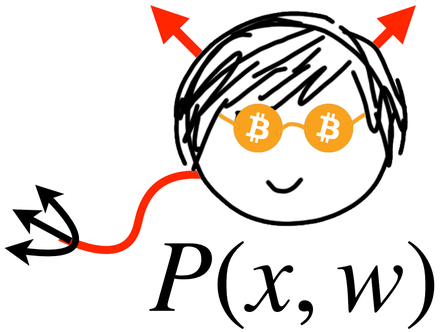
Trimming Resilience

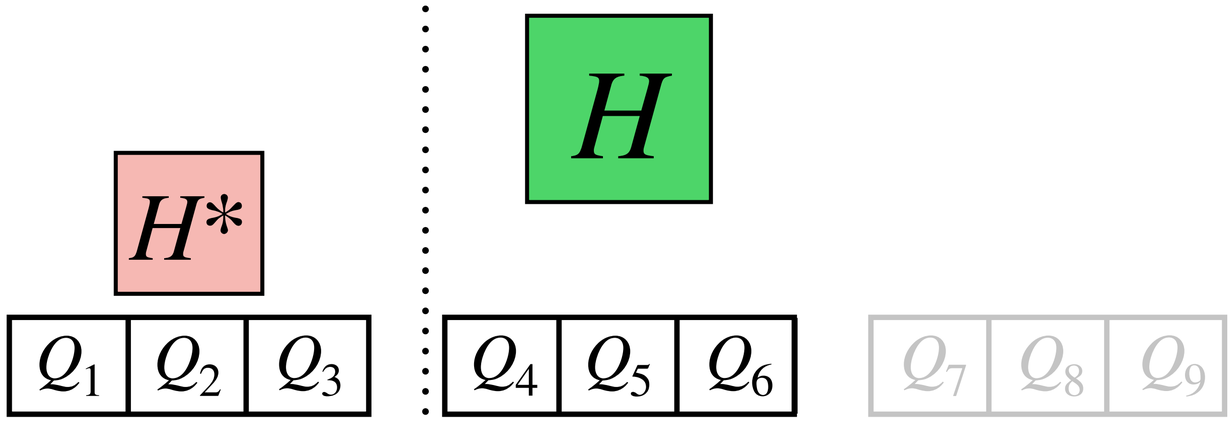




V checks at most $n = 2$ queries

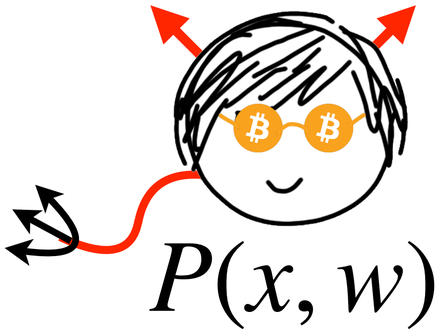
Trimming Resilience





V checks at most
$n = 2$ queries

Trimming Resilience



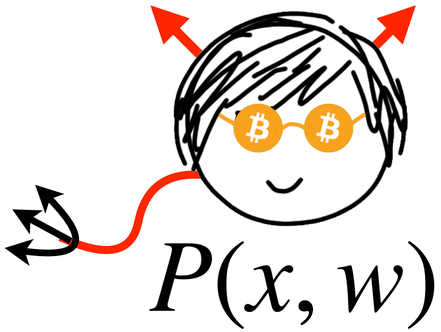
H^*

H

Q_1	Q_2	Q_3	Q_4	Q_5	Q_6	Q_7	Q_8	Q_9

V checks at most $n = 2$ queries

Trimming Resilience



$P(x, w)$

H^*

Q_1 Q_2 Q_3

H

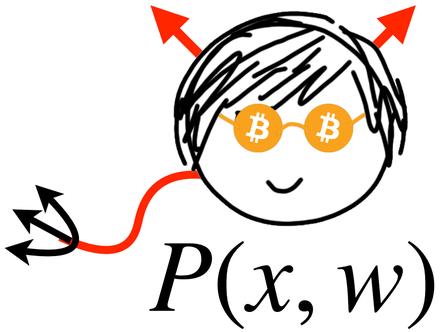
Q_4 Q_5 Q_6 Q_7 Q_8 Q_9

$V(x)$

H

V checks at most $n = 2$ queries

Trimming Resilience



$P(x, w)$

H^*

Q_1 Q_2 Q_3

H

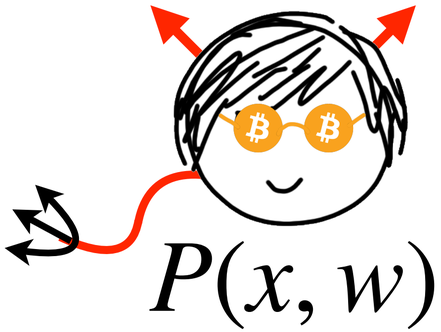
Q_4 Q_5 Q_6 Q_7 Q_8 Q_9

$V(x)$

H

V checks at most $n = 2$ queries

Trimming Resilience



$P(x, w)$

H^*

Q_1 Q_2 Q_3

H

Q_4 Q_5 Q_6 Q_7 Q_8 Q_9

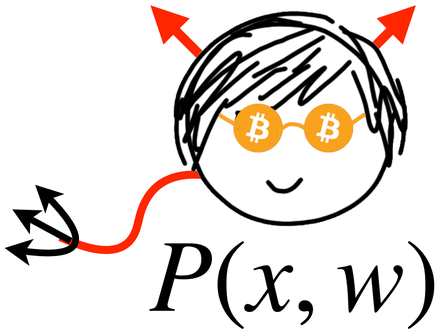
Reject

$V(x)$

H

V checks at most $n = 2$ queries

Trimming Resilience



$P(x, w)$

H^*

Q_1 Q_2 Q_3

H

Q_4 Q_5 Q_6

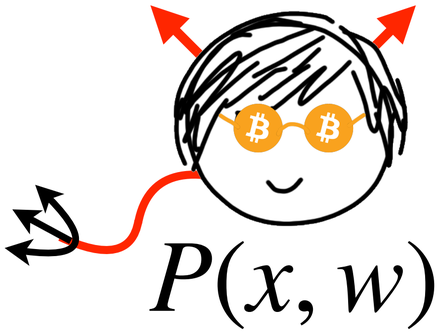
Q_7 Q_8 Q_9

$V(x)$

H

V checks at most $n = 2$ queries

Trimming Resilience



$P(x, w)$

H^*

Q_1 Q_2 Q_3

H

Q_4 Q_5 Q_6

Q_7 Q_8 Q_9

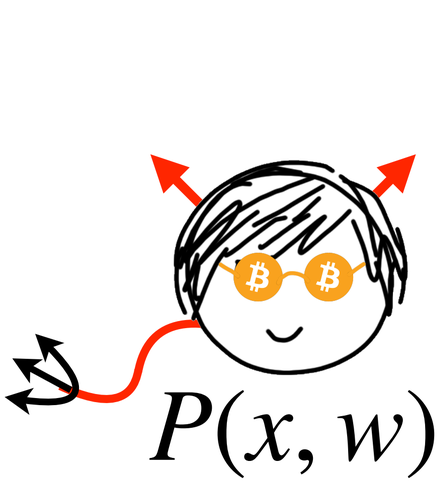
$V(x)$

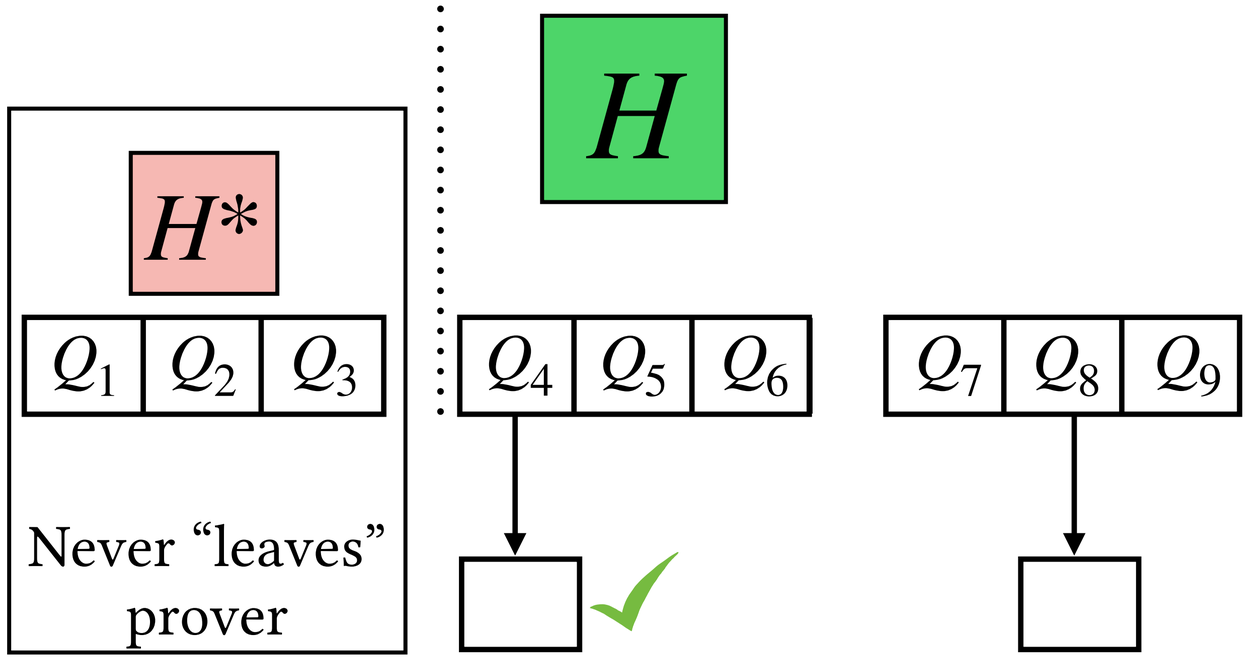
H

$\Pr[V \text{ accepts}] \geq 1/3$

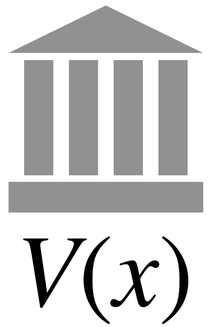
V checks at most $n = 2$ queries

Trimming Resilience





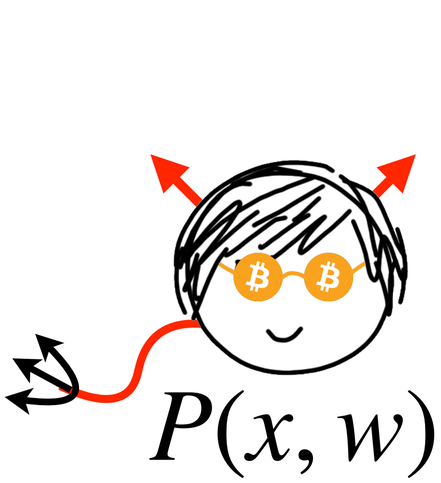
$\Pr[V \text{ accepts}] \geq 1/3$



H

V checks at most $n = 2$ queries

Trimming Resilience



$P(x, w)$

H^*

Q_1 Q_2 Q_3

Never "leaves" prover

H

Q_4 Q_5 Q_6 Q_7 Q_8 Q_9 Ext w

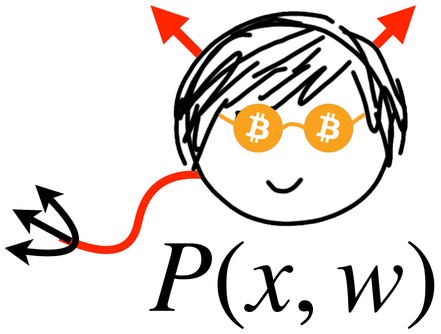
$\Pr[V \text{ accepts}] \geq 1/3$

$V(x)$

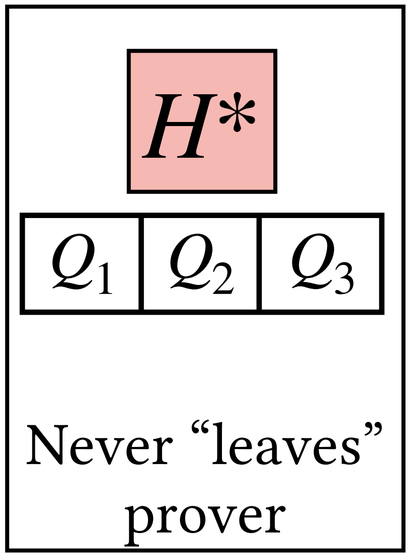
H

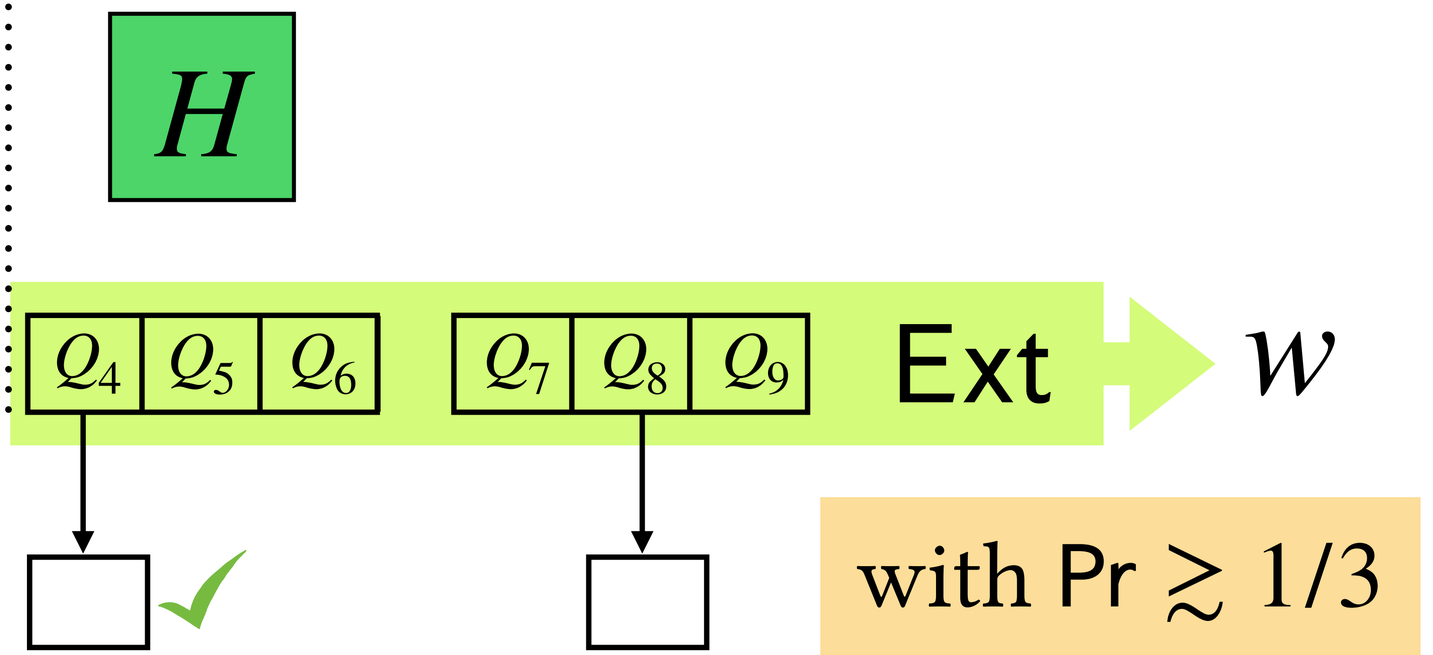
V checks at most $n = 2$ queries

Trimming Resilience



$P(x, w)$





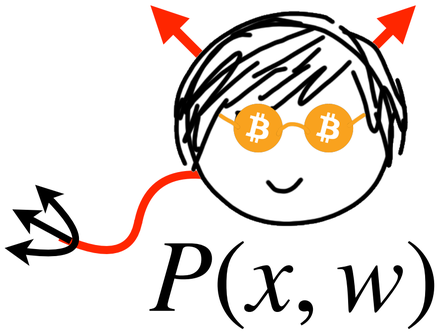
with Pr $\gtrsim 1/3$

Pr[V accepts] $\geq 1/3$

$V(x)$

V checks at most $n = 2$ queries

Trimming Resilience



$P(x, w)$

H^*

Q_7 Q_5 Q_3

Never "leaves" prover

H

(for any 3-partitioning)

Q_8 Q_1 Q_9 Q_2 Q_4 Q_6 Ext w

with Pr $\gtrsim 1/3$

Pr[V accepts] $\geq 1/3$

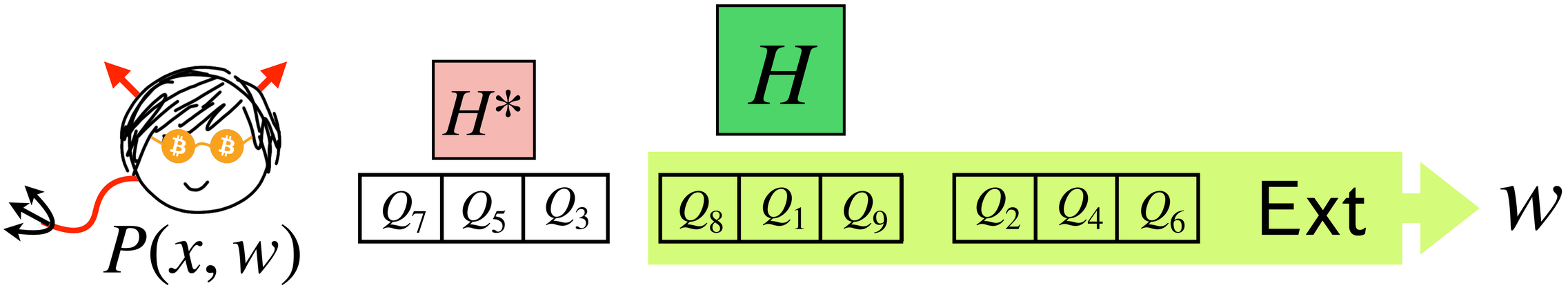
$V(x)$

H

V checks at most $n = 2$ queries

Trimming Resilience

Lemma: For any $n + 1$-partitioning of RO queries, omitting *one* partition still allows extraction

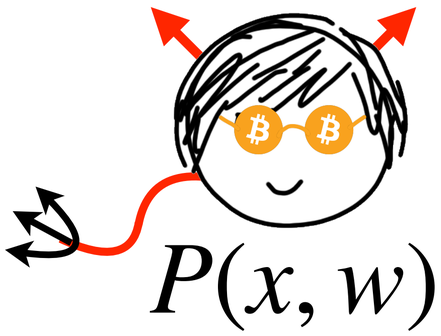


Trimming Resilience

Lemma: For any $n + 1$-partitioning of RO queries, omitting *one* partition still allows extraction

(random)

(w. noticeable probability)



$P(x, w)$

H^*

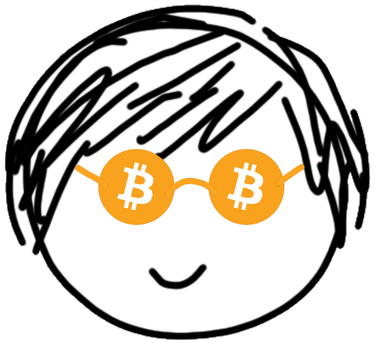
Q_7	Q_5	Q_3

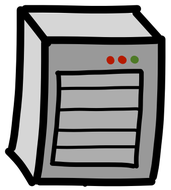
H

Q_8	Q_1	Q_9	Q_2	Q_4	Q_6

Ext → w

Oracle Respecting Distribution

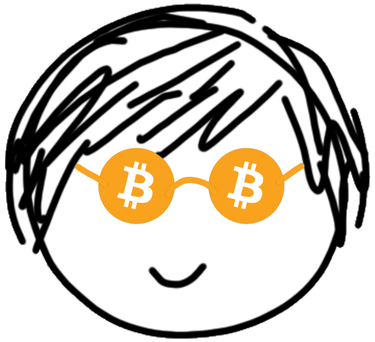




(x, w)

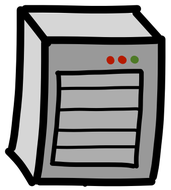
$w_0, w_1, w_2 \leftarrow \mathsf{Share}(w)$

Oracle Respecting Distribution



(x, w)

H



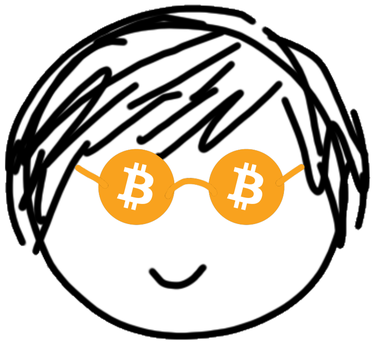
w_1

H

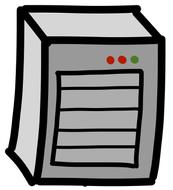
w_2

H

Oracle Respecting Distribution

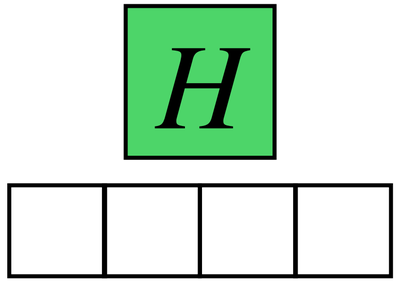


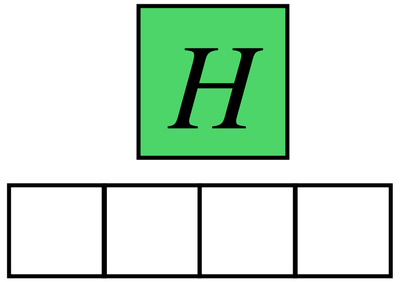
(x, w)

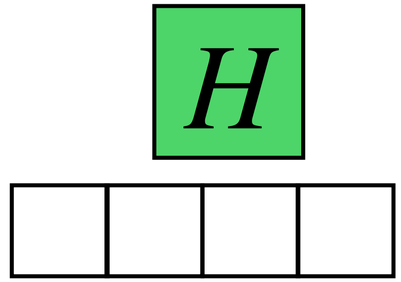


w_1

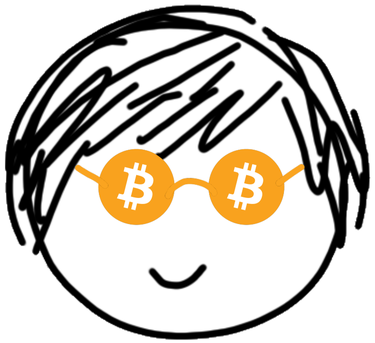
w_2



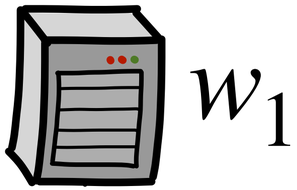


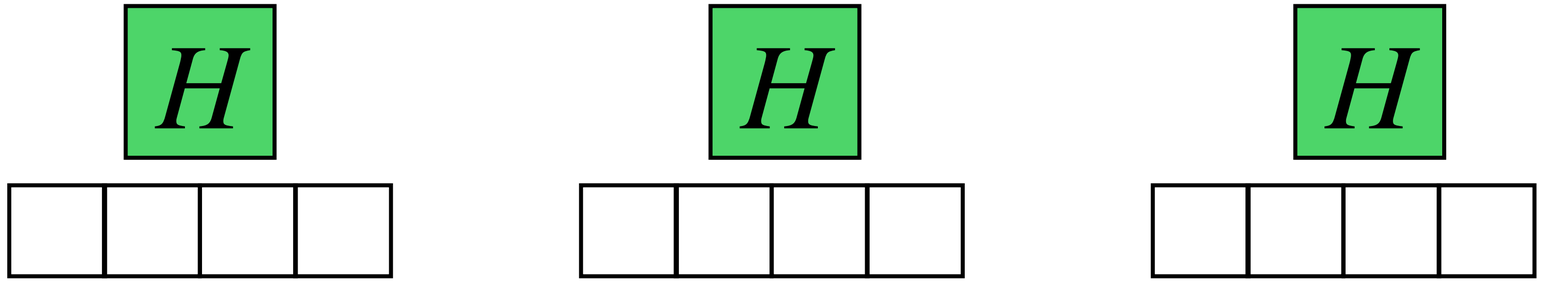


Oracle Respecting Distribution



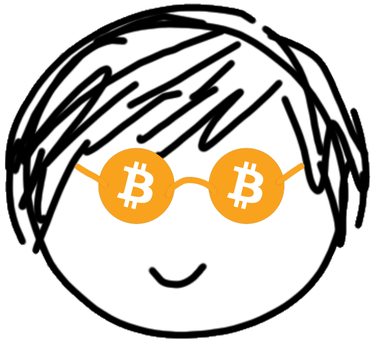
(x, w)



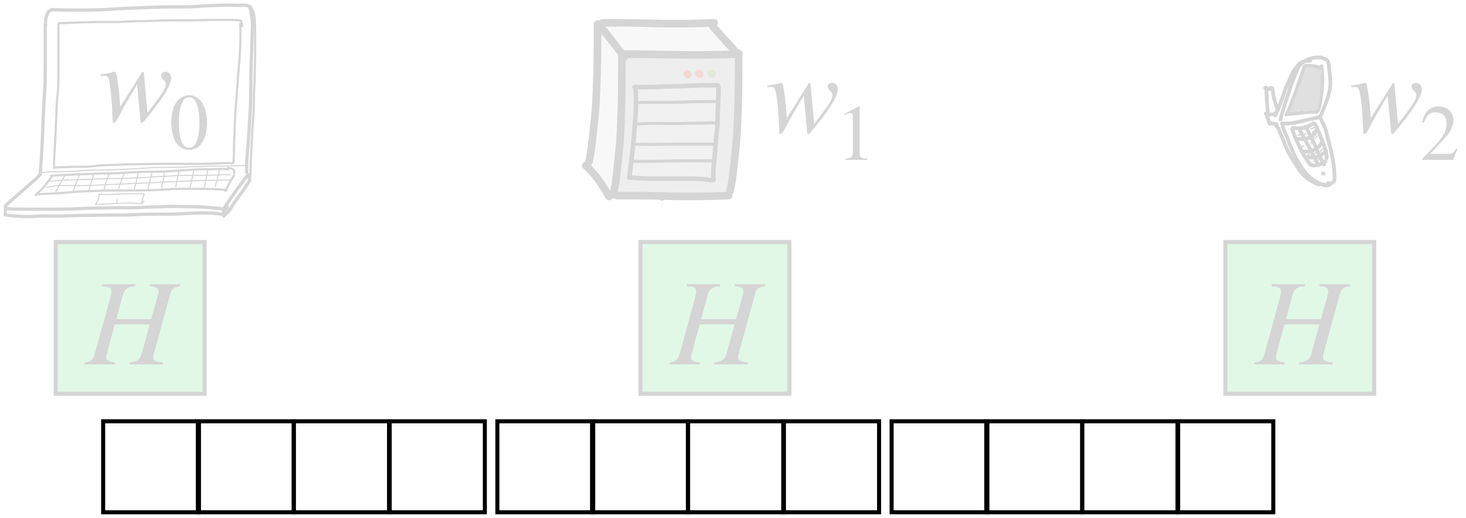


π

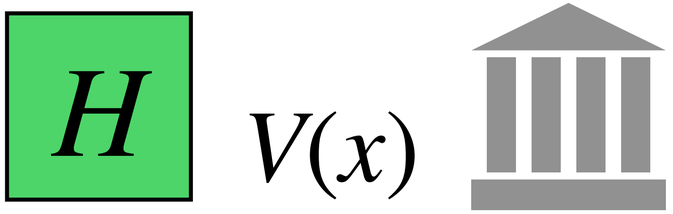
Oracle Respecting Distribution



(x, w)

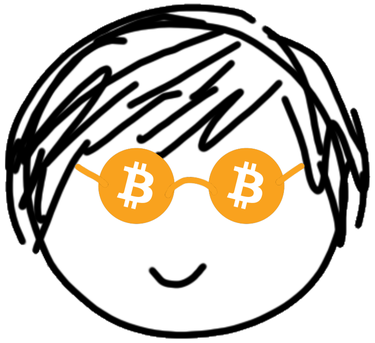


π

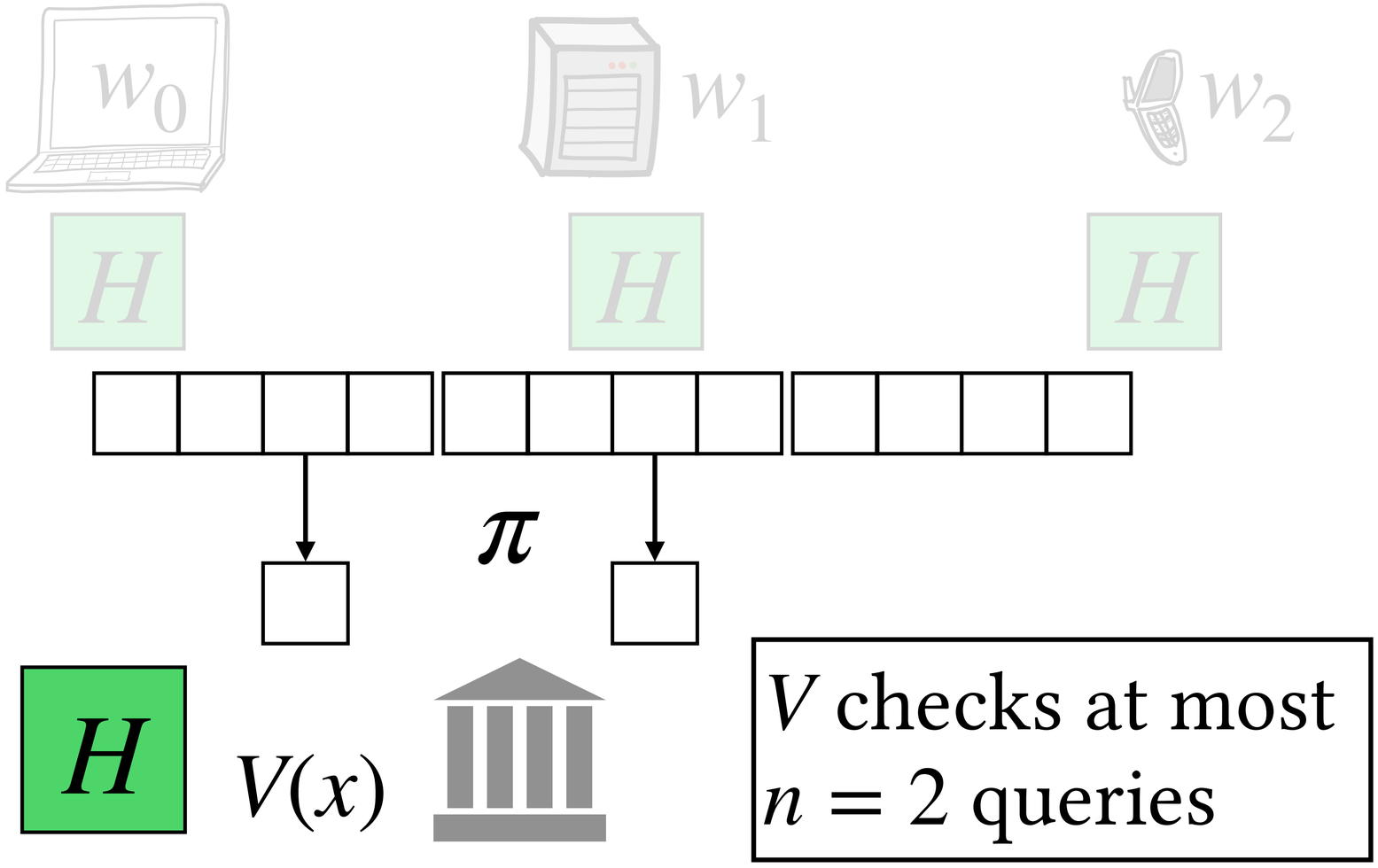


V checks at most $n = 2$ queries

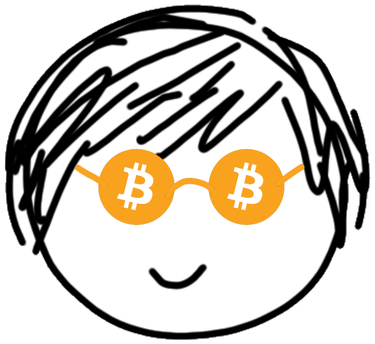
Oracle Respecting Distribution



(x, w)



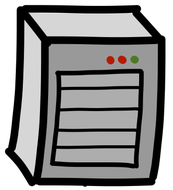
Oracle Respecting Distribution



(x, w)

Natural partitioning

w_0



w_1

w_2

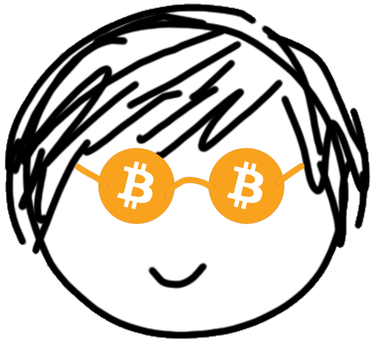
H H H

π

H $V(x)$

V checks at most $n = 2$ queries

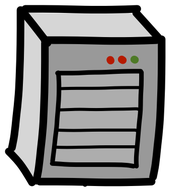
Oracle Respecting Distribution



(x, w)

Natural partitioning

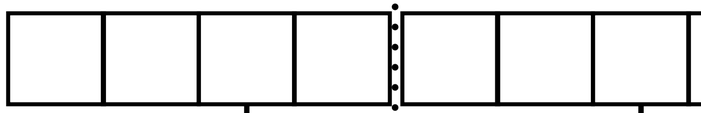
w_0



w_1

w_2

H H H^*

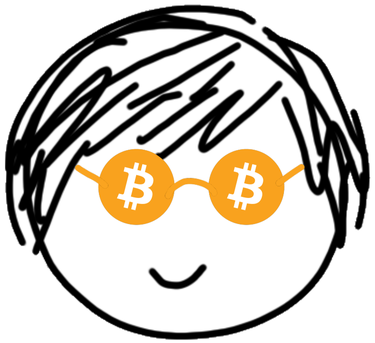


π

H $V(x)$

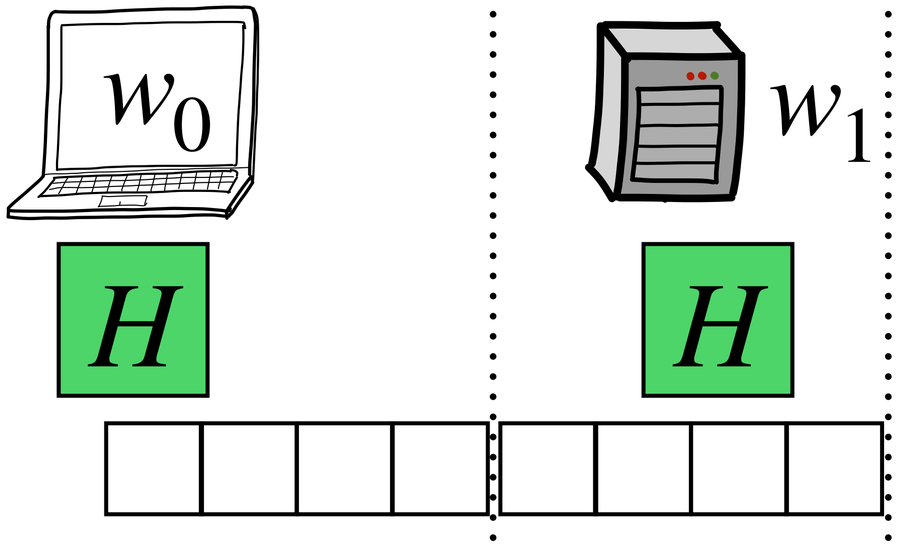
V checks at most $n = 2$ queries

Oracle Respecting Distribution

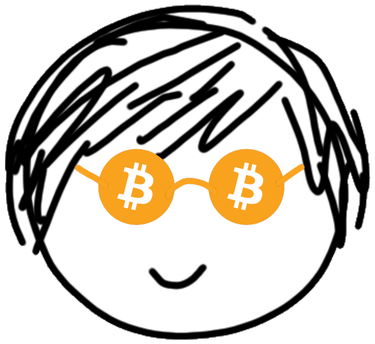


(x, w)

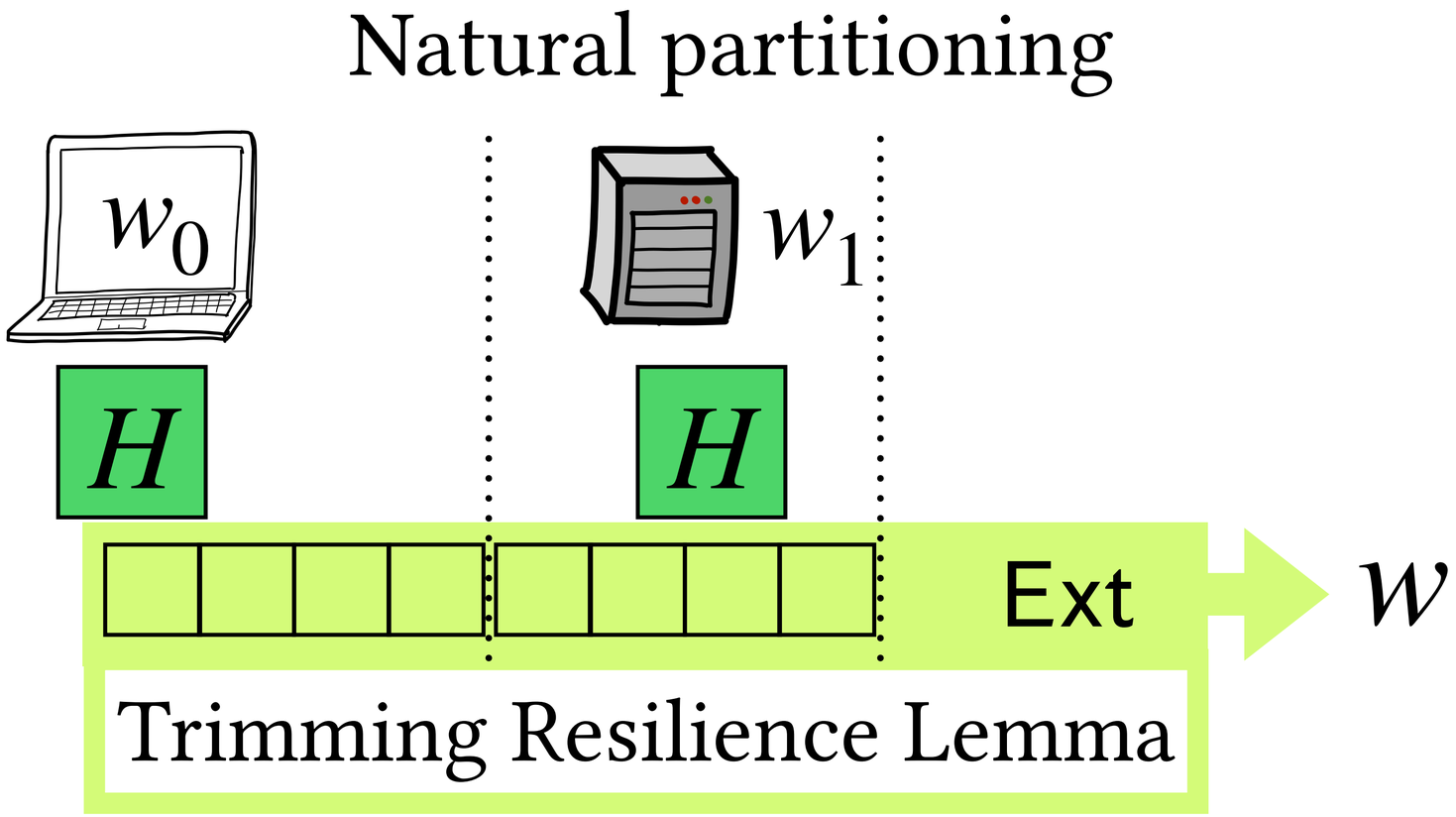
Natural partitioning



Oracle Respecting Distribution



(x, w)



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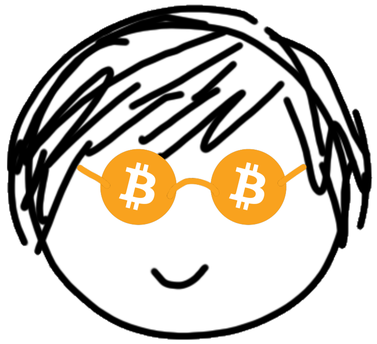
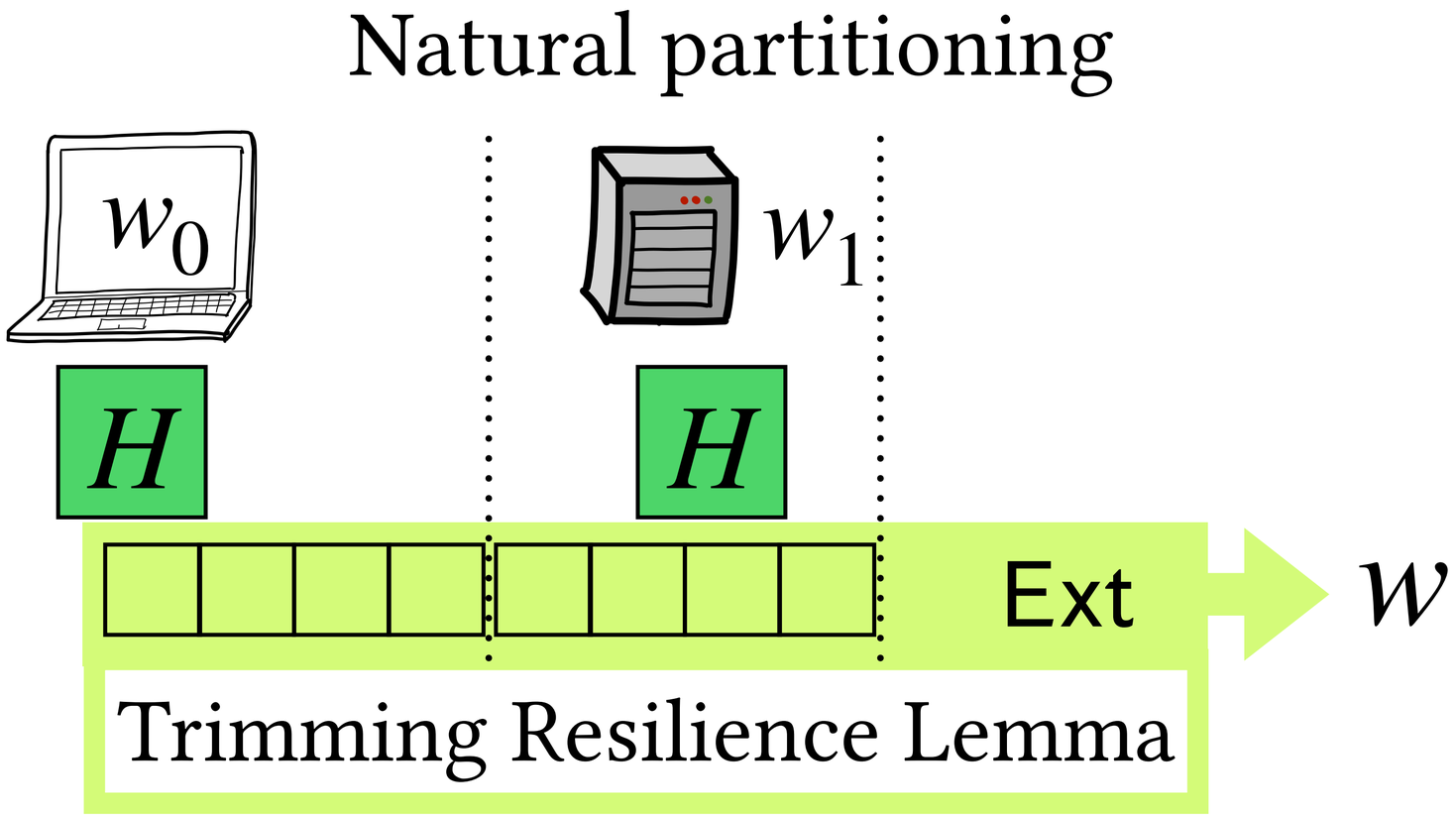
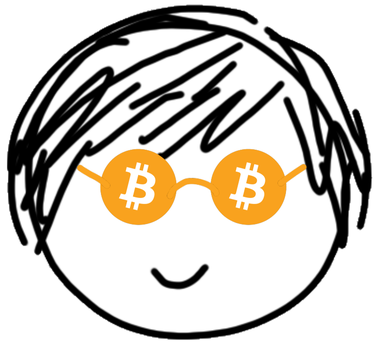


(x, w)

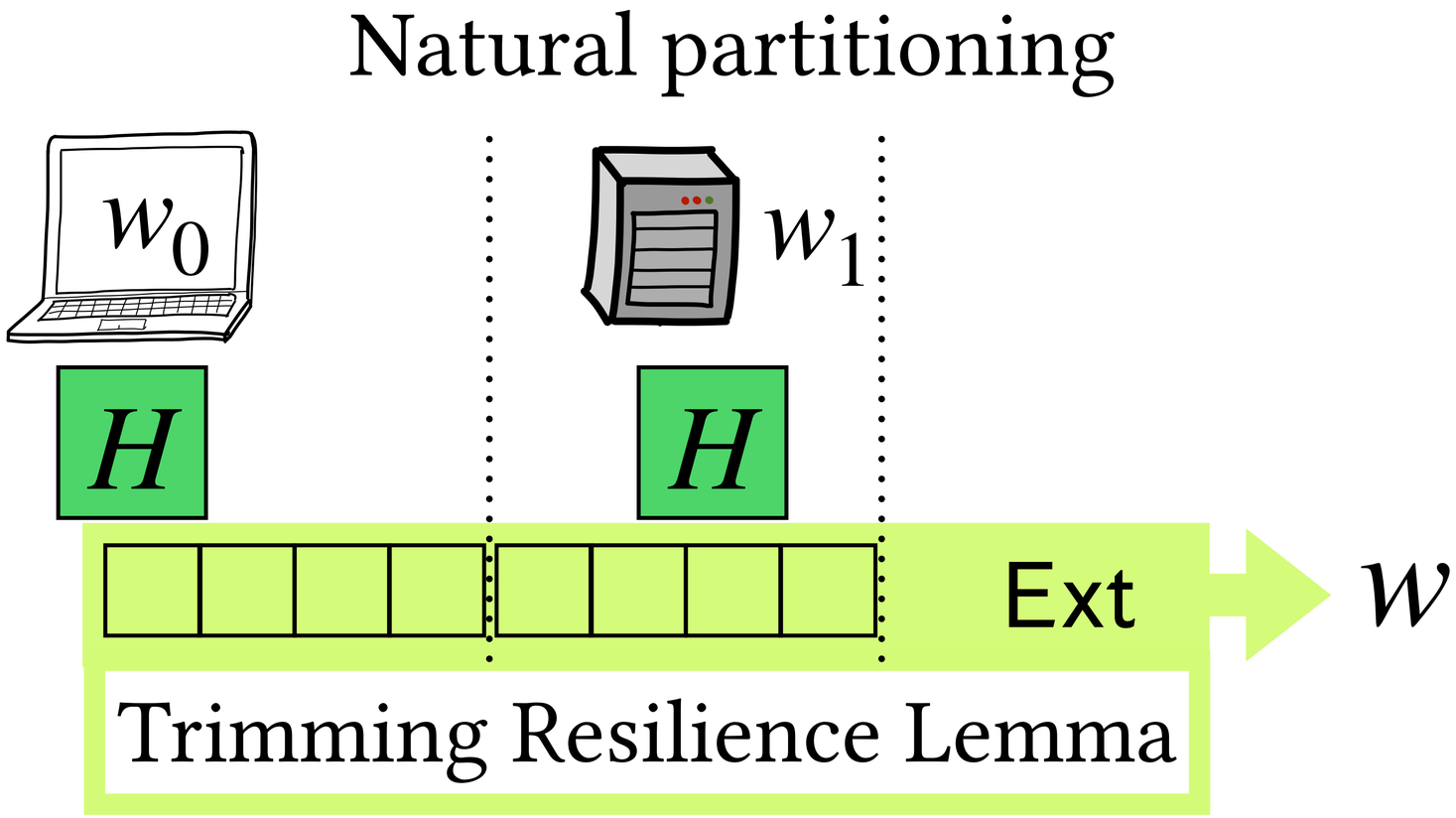


Two views are sufficient to reconstruct the witness

Oracle Respecting Distribution



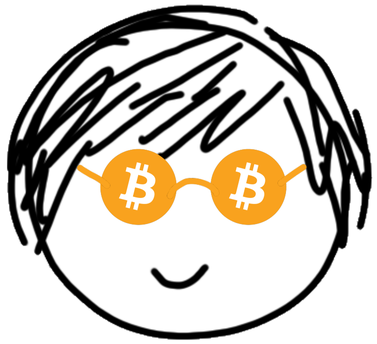
(x, w)



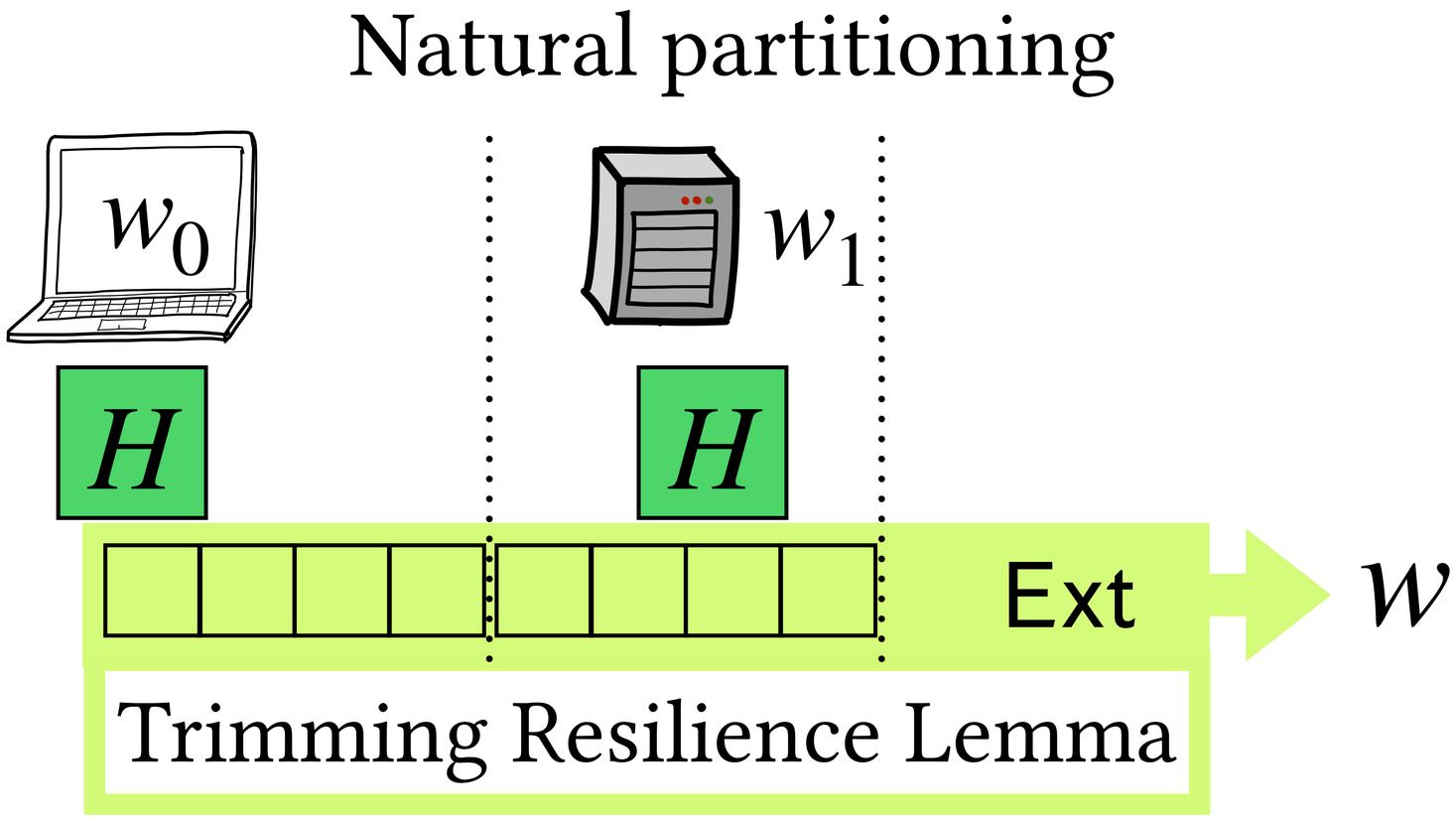
Two views are sufficient to reconstruct the witness

3 party ORD protocol can not withstand 2 passive corruptions

Oracle Respecting Distribution



(x, w)



Two views are sufficient to reconstruct the witness

n party ORD protocol can not withstand n-1 passive corruptions

Caveats

- The n-party protocol must be mapped to a single party algorithm to apply the trimming lemma
- This mapping induces one of two artefacts:
  - Protocol property: Each RO query in the protocol must "traceable" to the party that first made it

    OR
  - NIZK property: $\mathsf{Ext}(\overrightarrow{Q}, \pi)$ does not actually need $H(\overrightarrow{Q})$

Honest Majority?

- Previous technique can not be directly extended for fewer than $n - O(1)$ corruptions

  $\exists$ NIZKPoK of DLog π s.t. for any constant c, $\exists$ n-party ORD protocol to securely compute π with tolerance to $c \cdot n$ malicious corruptions
  Caveat: only beats trivial solution when $n > \kappa$

- However, ORD protocols for NIZKs where Ext needs a *single private query* of P seem unlikely **for even one corruption**

Conclusion

- We showed that n-party protocols to securely compute certain hash-based signatures/NIZKs can not make blackbox use of the same hash function
  - Includes MPC-in-the-head, Fischlin/Unruh/Pass/Ks22 transform, PCPs/IOPs
- Dist. NIZK Verifier must depend on #parties—could indicate that thresh. signature must grow with #signers?

Thanks!

Thanks **Eysa Lee** for