Threshold Schnorr with Stateless Deterministic Signing from Standard Assumptions

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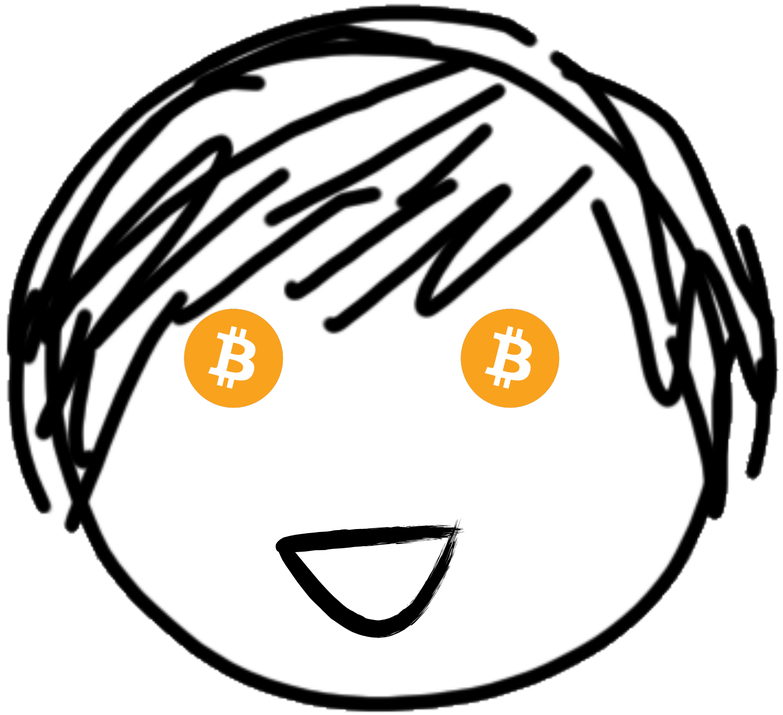

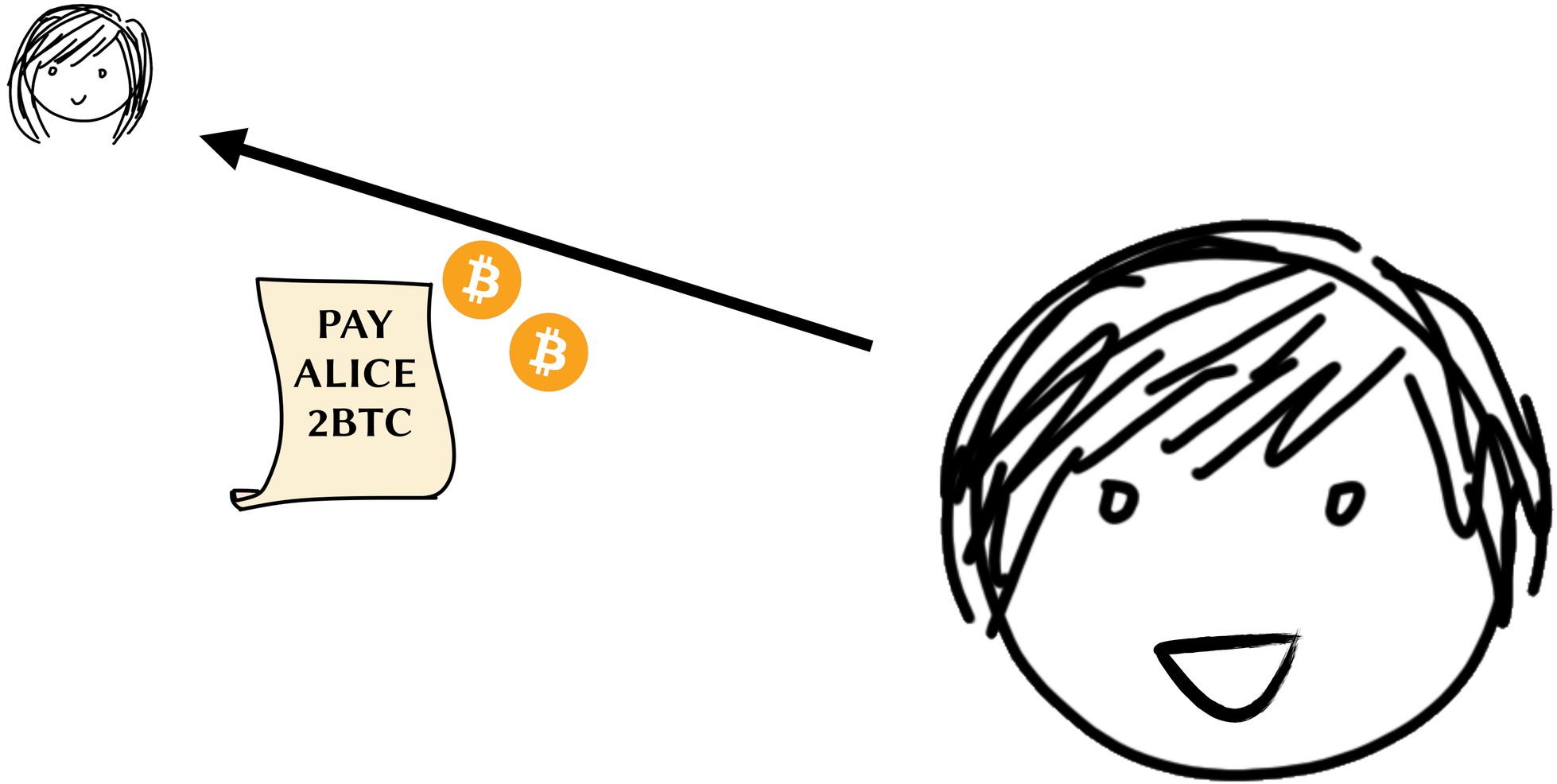
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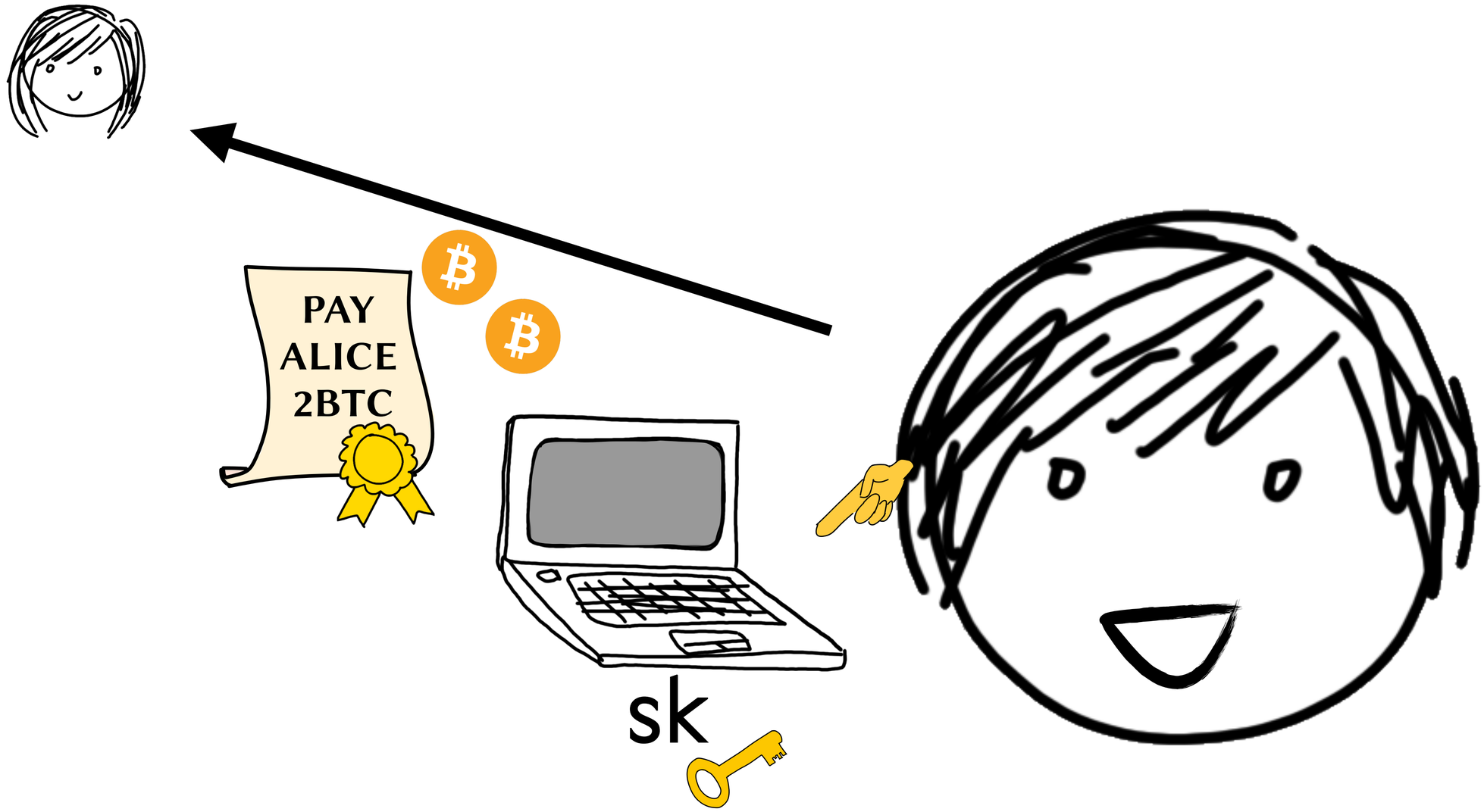
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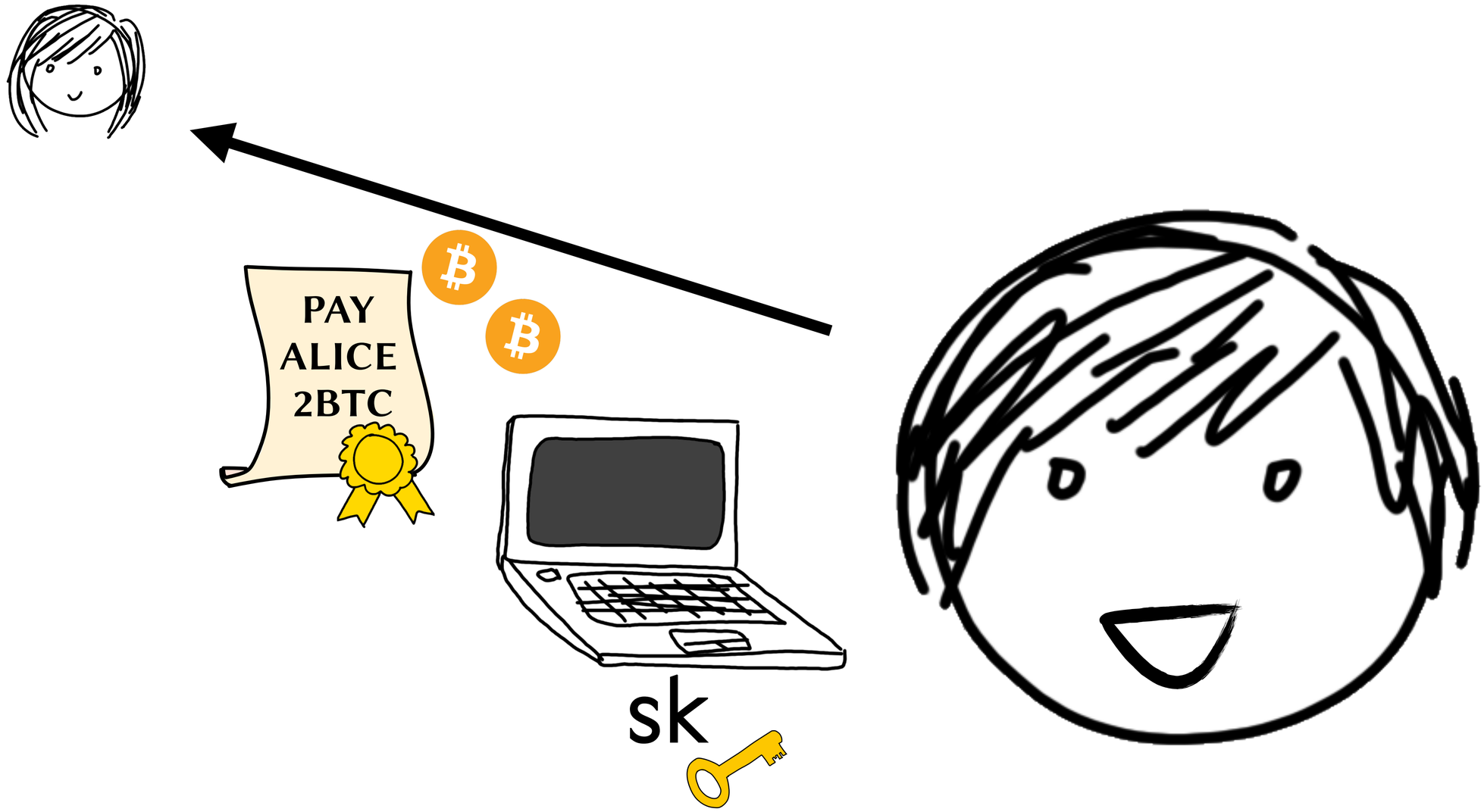
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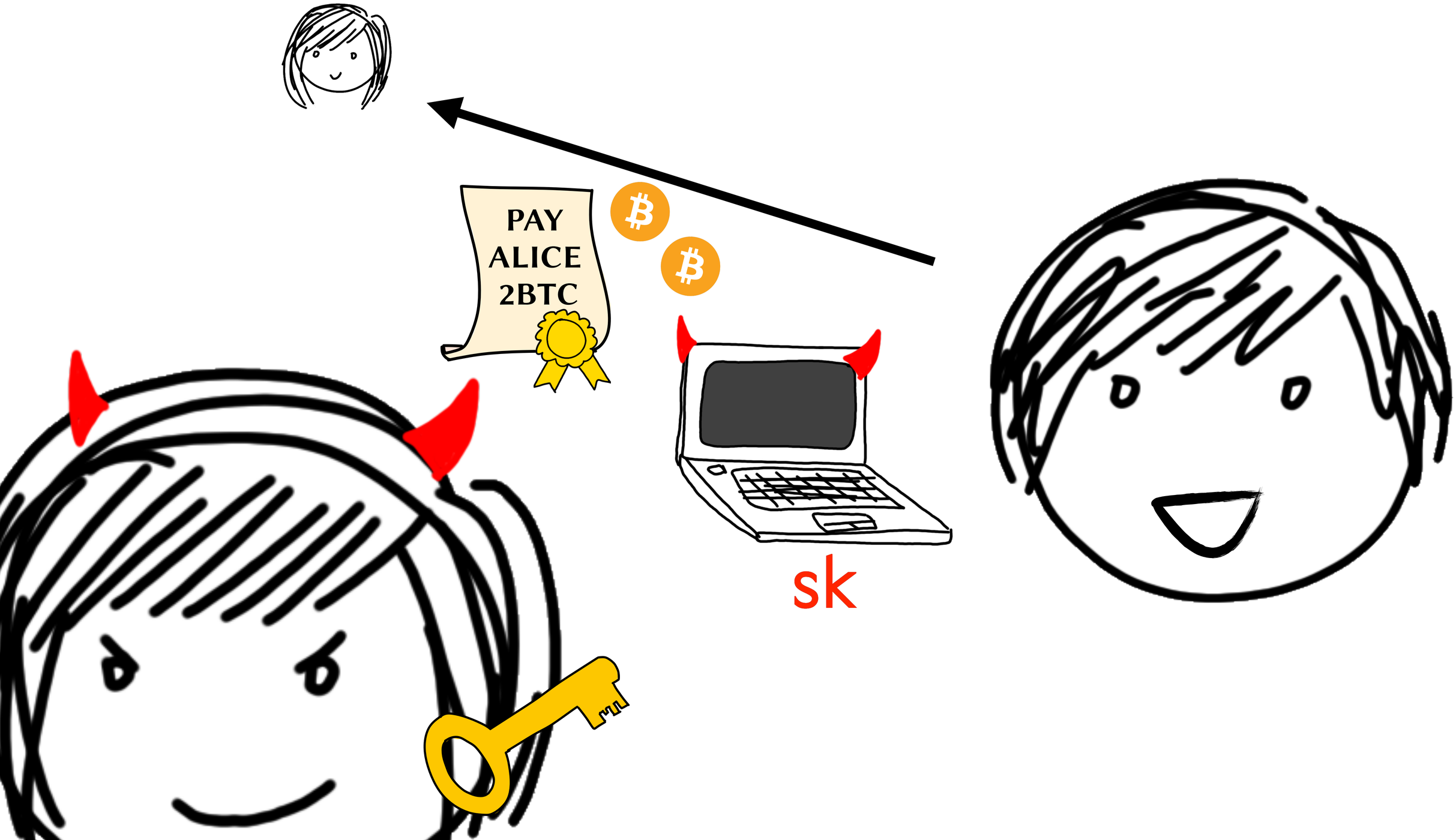
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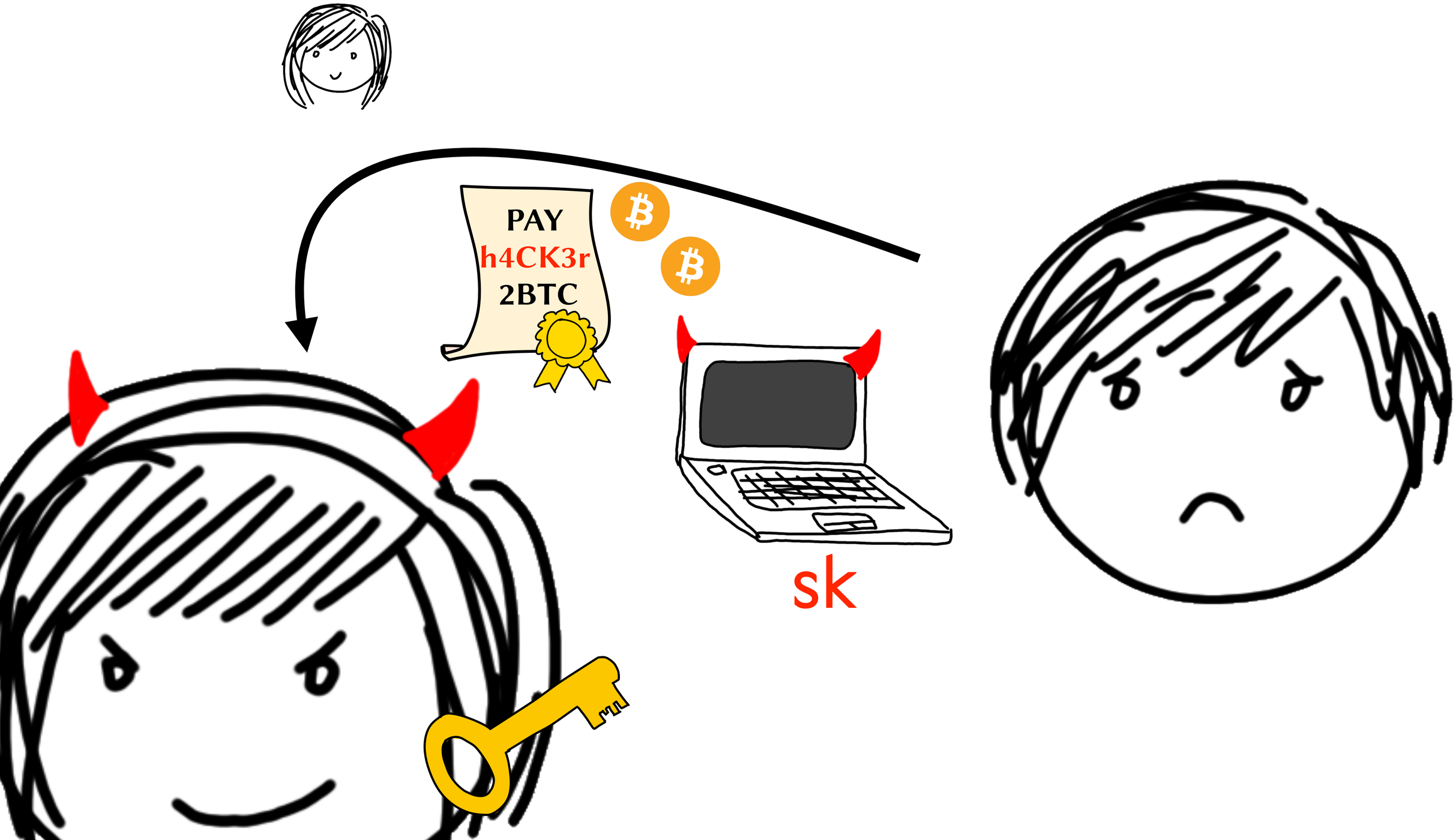
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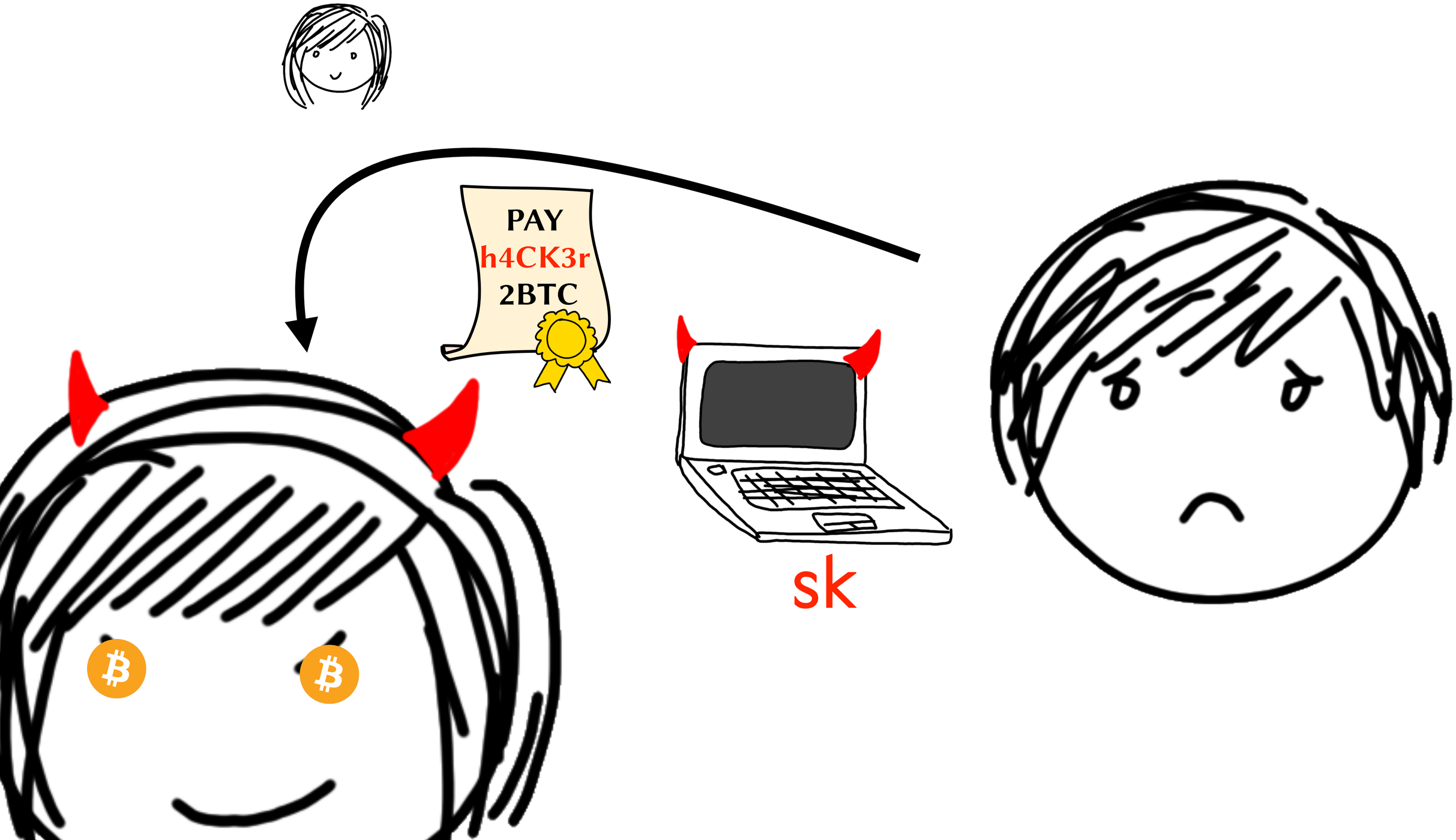
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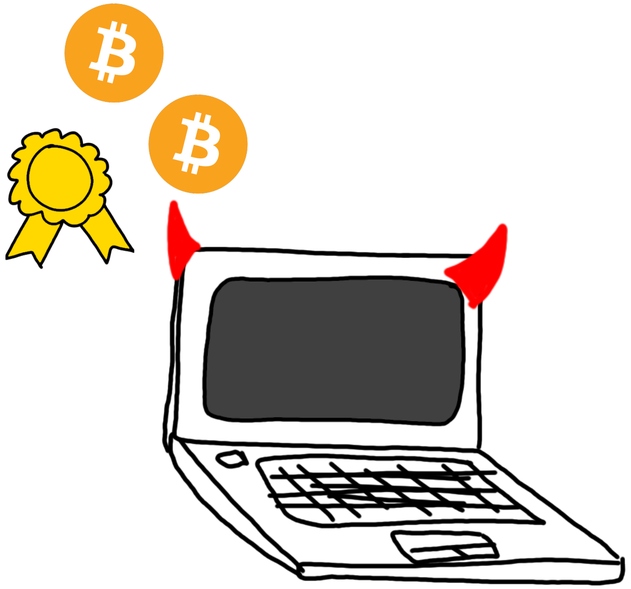
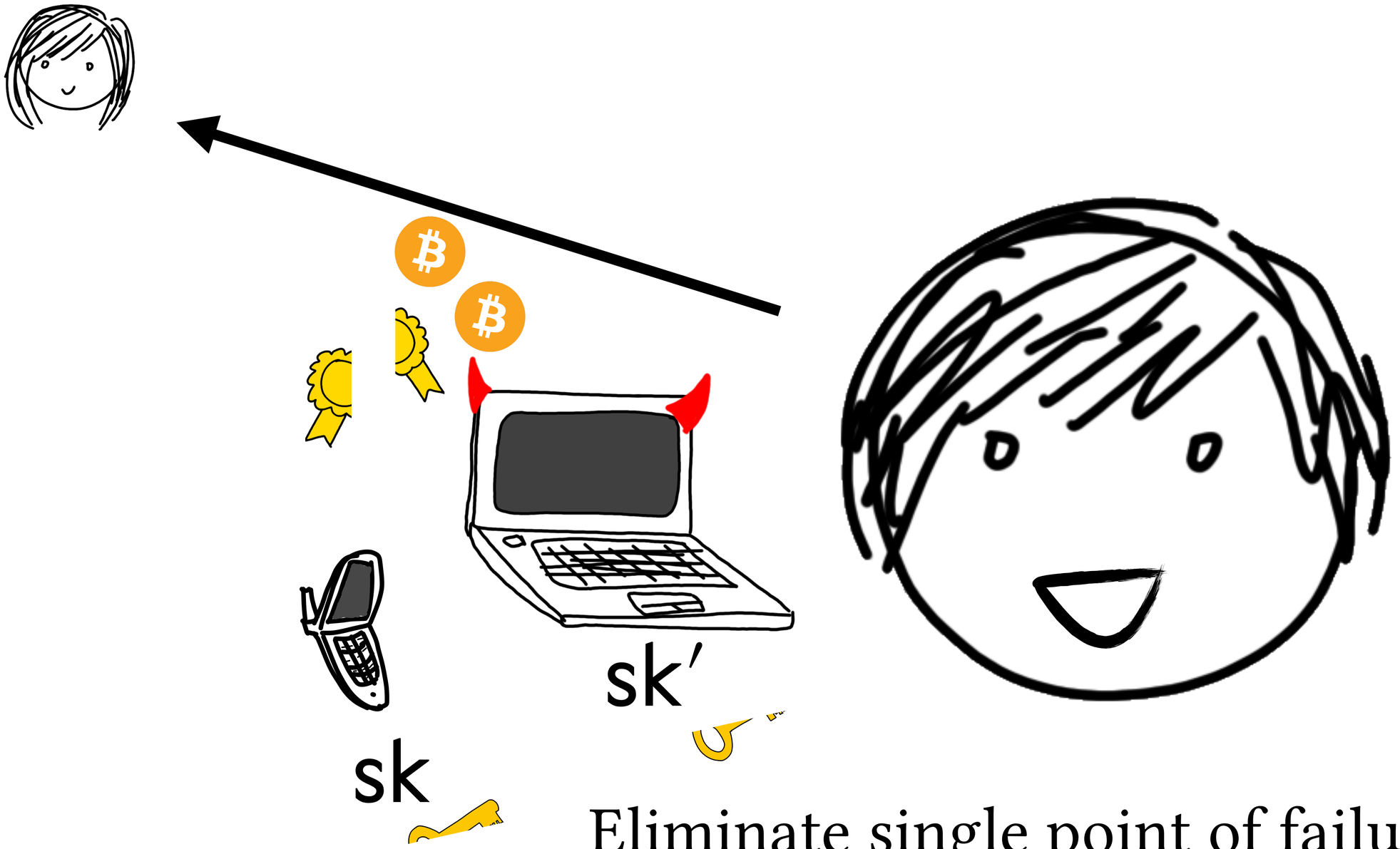
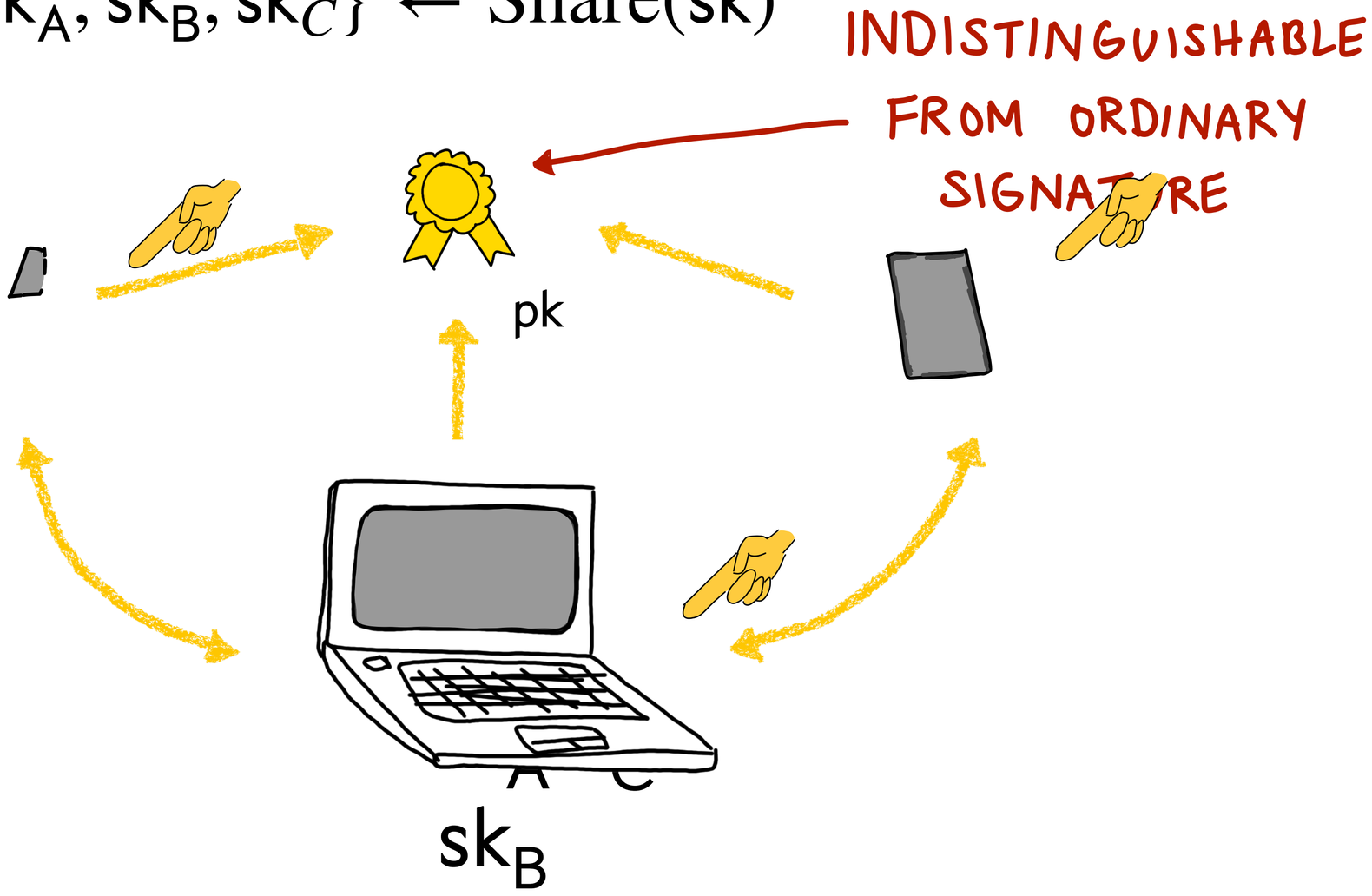




Eliminate single point of failure

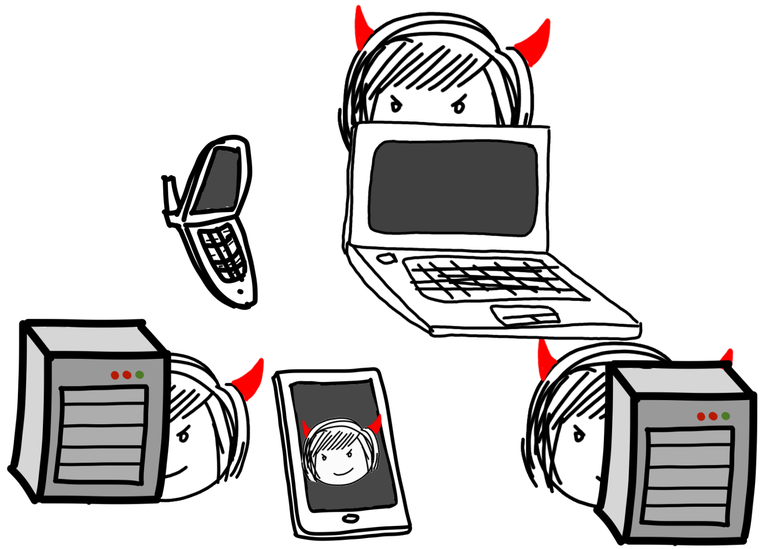
Threshold Signature

$$\{sk_A, sk_B, sk_C\} \leftarrow \text{Share}(sk)$$



Adversary Model

- Corruption threshold



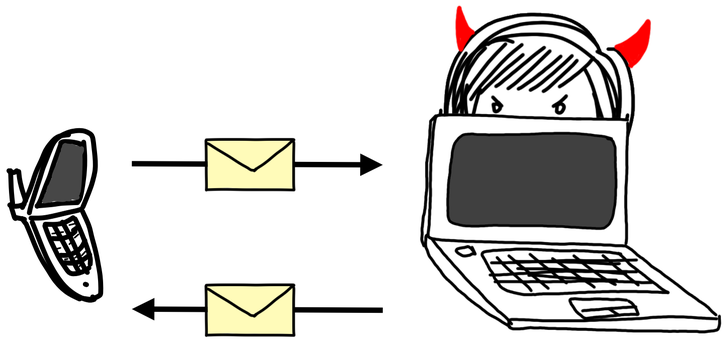
Dishonest majority
(only one device uncompromised)

Adversary Model

- Corruption threshold

Dishonest majority
(only one device uncompromised)

- Adversarial behaviour

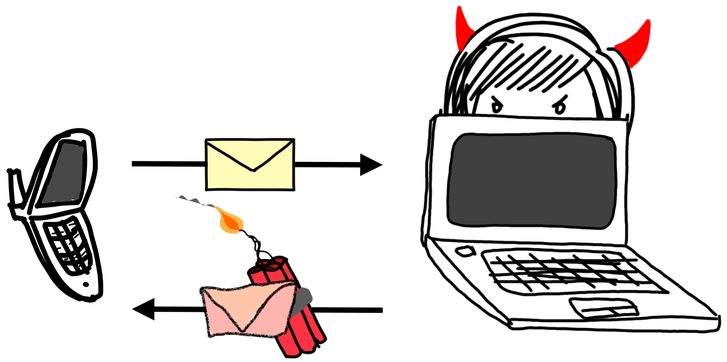


Adversary Model

- Corruption threshold

Dishonest majority
(only one device uncompromised)

- Adversarial behaviour



Malicious
(arbitrary deviations from protocol)

Schnorr's Signature Scheme

- Elegant signature scheme with security based on the hardness of computing discrete logarithms in carefully chosen group [Schnorr 89]
- Patent initially hampered widespread adoption
- Now seeing increased deployment across the internet in the form of EdDSA [Bernstein Duif Lange Schwabe Yang 11]
- Very easy to distribute with natural threshold key generation and signing protocols [Stinson Strobl 01][Gennaro Jarecki Krawczyk Rabin 03]

Schnorr Key Generation

$$\text{SchnorrKeyGen}(\mathbb{G}, G, q):$$

$$\mathsf{sk} \leftarrow \mathbb{Z}_q$$

$$\mathsf{PK} = \mathsf{sk} \cdot G$$

$$\text{output } (\mathsf{sk}, \mathsf{PK})$$

secret **k**ey: kept private

Public **K**ey: exposed to the outside world

Schnorr Signing

$\text{SchnorrKeyGen}(\mathbb{G}, G, q)$:

$\mathsf{sk} \leftarrow \mathbb{Z}_q$

$\mathsf{PK} = \mathsf{sk} \cdot G$

output $(\mathsf{sk}, \mathsf{PK})$

$\text{SchnorrSign}(\mathsf{sk}, m)$:

Schnorr Signing

SchnorrKeyGen$(\mathbb{G}, G, q)$:

$\mathsf{sk} \leftarrow \mathbb{Z}_q$

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output $(\mathsf{sk}, \mathsf{PK})$

SchnorrSign(sk, m) :

$k \leftarrow \mathbb{Z}_q$

Schnorr Signing

$\text{SchnorrKeyGen}(\mathbb{G}, G, q)$:

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output $(\mathsf{sk}, \mathsf{PK})$

$\text{SchnorrSign}(\mathsf{sk}, m)$:

$k \leftarrow \mathbb{Z}_q$

$R = k \cdot G$

NONCE
One-time use value

Schnorr Signing

$\text{SchnorrKeyGen}(\mathbb{G}, G, q)$:

$\mathsf{sk} \leftarrow \mathbb{Z}_q$

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output $(\mathsf{sk}, \mathsf{PK})$

$\text{SchnorrSign}(\mathsf{sk}, m)$:

$k \leftarrow \mathbb{Z}_q$

$R = k \cdot G$

$e = H(R \| m)$

NONCE
One-time use value

Schnorr Signing

$\text{SchnorrKeyGen}(\mathbb{G}, G, q)$:

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$\text{SchnorrSign}(\mathsf{sk}, m)$:

$k \leftarrow \mathbb{Z}_q$

$R = k \cdot G$

NONCE
One-time use value

$e = H(R \| m)$

$s = k - \mathsf{sk} \cdot e \pmod{q}$

Schnorr Signing

$\text{SchnorrKeyGen}(\mathbb{G}, G, q)$:

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$R = k \cdot G$

NONCE
One-time use value

$e = H(R \| m)$

$s = k - \mathsf{sk} \cdot e \pmod{q}$

$\sigma = (s, R)$

output σ

Schnorr Signing

SchnorrKeyGen$(\mathbb{G}, G, q)$:

$\mathsf{sk} \leftarrow \mathbb{Z}_q$

$\mathsf{PK} = \mathsf{sk} \cdot G$

output $(\mathsf{sk}, \mathsf{PK})$

SchnorrSign(sk, m) :

$k \leftarrow \mathbb{Z}_q$

$R = k \cdot G$

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One-time use value

$e = H(R \| m)$

$s = k - \mathsf{sk} \cdot e \pmod{q}$

$\sigma = (s, R)$

output σ

Verifying a signature: $s \cdot G \stackrel{?}{=} R - e \cdot \mathsf{PK}$

Schnorr Signing

SchnorrKeyGen$(\mathbb{G}, G, q)$:

$\mathsf{sk} \leftarrow \mathbb{Z}_q$

$\mathsf{PK} = \mathsf{sk} \cdot G$

output $(\mathsf{sk}, \mathsf{PK})$

SchnorrSign(sk, m) :

$k \leftarrow \mathbb{Z}_q$

$R = k \cdot G$

NONCE
One-time use value

$e = H(R \| m)$

$s = k - \mathsf{sk} \cdot e \pmod{q}$

$\sigma = (s, R)$

output σ

Verifying a signature: $s \cdot G \stackrel{?}{=} R - e \cdot \mathsf{PK}$

$k \cdot G$ $\quad$ $\mathsf{sk} \cdot G$

Schnorr Signing

SchnorrKeyGen$(\mathbb{G}, G, q)$:

$\mathsf{sk} \leftarrow \mathbb{Z}_q$

$\mathsf{PK} = \mathsf{sk} \cdot G$

output $(\mathsf{sk}, \mathsf{PK})$

SchnorrSign(sk, m) :

$k \leftarrow \mathbb{Z}_q$

$R = k \cdot G$

NONCE
One-time use value

$e = H(R \| m)$

$s = k - \mathsf{sk} \cdot e \pmod{q}$

$\sigma = (s, R)$

output σ

Verifying a signature:

$s \cdot G \stackrel{?}{=} R - e \cdot \mathsf{PK}$

$k \cdot G$ $\quad$ $\mathsf{sk} \cdot G$

$s \cdot G \stackrel{?}{=} (k - e \cdot \mathsf{sk}) \cdot G$

Schnorr Signing

$$\text{SchnorrSign}(\mathsf{sk}, m):$$

$$k \leftarrow \mathbb{Z}_q$$

$$R = k \cdot G$$

$$e = H(R \| m)$$

$$s = k - \mathsf{sk} \cdot e$$

$$\sigma = (s, R)$$

$$\text{output } \sigma$$

Schnorr Signing

$\text{SchnorrSign}(\mathsf{sk}, m)$:

$k \leftarrow \mathbb{Z}_q$

$R = k \cdot G$

$e = H(R \| m)$

$s = k - \mathsf{sk} \cdot e$

$\sigma = (s, R)$

output σ

Linear function of k, sk
Linear operations are very easy to distribute with most natural secret sharing schemes

Distributing Schnorr Signing

SchnorrSign(sk, m) :

$k \leftarrow \mathbb{Z}_q$

$R = k \cdot G$

$e = H(R \| m)$

$s = k - \mathsf{sk} \cdot e$

$\sigma = (s, R)$

output σ

$\mathsf{sk}_A + \mathsf{sk}_B = \mathsf{sk}$

Distributing Schnorr Signing

SchnorrSign(sk, m) :

$k \leftarrow \mathbb{Z}_q$

$R = k \cdot G$

$e = H(R \| m)$

$s = k - \mathsf{sk} \cdot e$

$\sigma = (s, R)$

output σ

sk_A

sk_B

$k_A + k_B = k$

Distributing Schnorr Signing

sk_A

sk_B

SchnorrSign(sk, m) :

$k \leftarrow \mathbb{Z}_q$	$k_A \leftarrow \mathbb{Z}_q$	$k_B \leftarrow \mathbb{Z}_q$
$R = k \cdot G$	$R_A = k_A \cdot G$	$R_B = k_B \cdot G$

$e = H(R \| m)$

$s = k - \mathsf{sk} \cdot e$

$\sigma = (s, R)$

output σ

Distributing Schnorr Signing

sk_A

sk_B

SchnorrSign(sk, m) :

$k \leftarrow \mathbb{Z}_q$

$R = k \cdot G$

$e = H(R \| m)$

$s = k - \mathsf{sk} \cdot e$

$\sigma = (s, R)$

output σ

$k_A \leftarrow \mathbb{Z}_q$

$R_A = k_A \cdot G$

$k_B \leftarrow \mathbb{Z}_q$

$R_B = k_B \cdot G$

R_B

R_A

Distributing Schnorr Signing

sk_A

sk_B

SchnorrSign(sk, m) :

$k \leftarrow \mathbb{Z}_q$	$k_A \leftarrow \mathbb{Z}_q$	$k_B \leftarrow \mathbb{Z}_q$
$R = k \cdot G$	$R = R_A + R_B$	$R = R_A + R_B$
$e = H(R \Vert m)$		
$s = k - \mathsf{sk} \cdot e$		
$\sigma = (s, R)$		
output σ		

Distributing Schnorr Signing

sk_A

sk_B

SchnorrSign(sk, m) :

$k \leftarrow \mathbb{Z}_q$

$R = k \cdot G$

$e = H(R \| m)$

$s = k - \mathsf{sk} \cdot e$

$\sigma = (s, R)$

output σ

$k_A \leftarrow \mathbb{Z}_q$

$R = R_A + R_B$

$e = H(R \| m)$

$k_B \leftarrow \mathbb{Z}_q$

$R = R_A + R_B$

$e = H(R \| m)$

Distributing Schnorr Signing

sk_A

sk_B

SchnorrSign(sk, m) :

$k \leftarrow \mathbb{Z}_q$

$R = k \cdot G$

$e = H(R \| m)$

$s = k - \mathsf{sk} \cdot e$

$\sigma = (s, R)$

output σ

$k_A \leftarrow \mathbb{Z}_q$

$R = R_A + R_B$

$e = H(R \| m)$

$s_A = k_A - \mathsf{sk}_A \cdot e$

$k_B \leftarrow \mathbb{Z}_q$

$R = R_A + R_B$

$e = H(R \| m)$

$s_B = k_B - \mathsf{sk}_B \cdot e$

Distributing Schnorr Signing

SchnorrSign(sk, m) :

$k \leftarrow \mathbb{Z}_q$

$R = k \cdot G$

$e = H(R \| m)$

$s = k - \mathsf{sk} \cdot e$

$\sigma = (s, R)$

output σ

sk_A

sk_B

$k_A \leftarrow \mathbb{Z}_q$

$R = R_A + R_B$

$e = H(R \| m)$

$s_A = k_A - \mathsf{sk}_A \cdot e$

$k_B \leftarrow \mathbb{Z}_q$

$R = R_A + R_B$

$e = H(R \| m)$

$s_B = k_B - \mathsf{sk}_B \cdot e$

s_B

s_A

Distributing Schnorr Signing

SchnorrSign(sk, m) :

$k \leftarrow \mathbb{Z}_q$

$R = k \cdot G$

$e = H(R \| m)$

$s = k - \mathsf{sk} \cdot e$

$\sigma = (s, R)$

output σ

sk_A

sk_B

$k_A \leftarrow \mathbb{Z}_q$	$k_B \leftarrow \mathbb{Z}_q$
$R = R_A + R_B$ ⟷	⟷ $R = R_A + R_B$
$e = H(R \| m)$	$e = H(R \| m)$
$s_A = k_A - \mathsf{sk}_A \cdot e$	$s_B = k_B - \mathsf{sk}_B \cdot e$
$s = s_A + s_B$ ⟷	⟷ $s = s_A + s_B$

Distributing Schnorr Signing

SchnorrSign(sk, m) :

$k \leftarrow \mathbb{Z}_q$

$R = k \cdot G$

$e = H(R \| m)$

$s = k - \mathsf{sk} \cdot e$

$\sigma = (s, R)$

output σ

sk_A

sk_B

$k_A \leftarrow \mathbb{Z}_q$	$k_B \leftarrow \mathbb{Z}_q$
$R = R_A + R_B$ ⟷	⟷ $R = R_A + R_B$
$e = H(R \| m)$	$e = H(R \| m)$
$s_A = k_A - \mathsf{sk}_A \cdot e$	$s_B = k_B - \mathsf{sk}_B \cdot e$
$s = s_A + s_B$ ⟷	⟷ $s = s_A + s_B$

Generalizes to n parties easily

Practical Issue

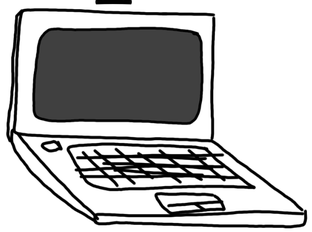
- Each Schnorr signature requires a fresh, one-time nonce (k, R)
- Security is extremely sensitive to the distribution of k — even a tiny amount of non-uniformity across signatures can be leveraged to retrieve sk in its entirety [Boneh Venkatesan 96][Howgrave-Graham Smart 01][Bleichenbacher 00] [Aranha Novaes Takahashi Tibouchi Yarom 20][Albrecht Heninger 21]
- Major concern in practice: "true" randomness is a scarce resource
  - Errors in implementation
  - Poorly seeded Random Number Generators

Cryptographic Solution

- This systems-level problem can be avoided with a simple cryptographic trick
  - During (one-time) secret key generation: sample a seed $\mathsf{sd} \leftarrow \{0,1\}^\kappa$
  - When signing m: instead of sampling fresh k, compute $k = F(\mathsf{sd}, m)$
  - Assuming $F : \{0,1\}^\kappa \to \mathbb{Z}_q$ is a pseudorandom function, the distribution of $F(\mathsf{sd}, m)$ is negligibly different from that of uniform choice of k per message
- This is a classic idea [Barwood 97][M'Raïhi Naccache Pointcheval Vaudenay 98] [Wigley 97] that is employed by the modern **EdDSA** variant of Schnorr

A Simple Attempt

sk_A



sk_B

$k_A \leftarrow \mathbb{Z}_q$

$k_B \leftarrow \mathbb{Z}_q$

$R_A = k_A \cdot G$

$R_B = k_B \cdot G$

$R = R_A + R_B$ ⟷ $R = R_A + R_B$

$e = H(R \| m)$

$e = H(R \| m)$

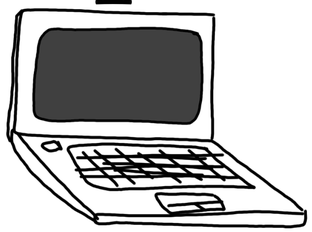
$s_A = k_A - \mathsf{sk}_A \cdot e$

$s_B = k_B - \mathsf{sk}_B \cdot e$

$s = s_A + s_B$ ⟷ $s = s_A + s_B$

A Simple Attempt

sk_A sd_A sd_B sk_B



$k_A \leftarrow \mathbb{Z}_q$

$R_A = k_A \cdot G$

$R = R_A + R_B$ ⟷ $R = R_A + R_B$

$e = H(R \| m)$ $\quad$ $e = H(R \| m)$

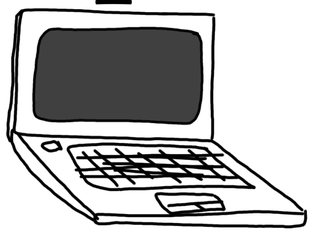
$s_A = k_A - \mathsf{sk}_A \cdot e$ $\quad$ $s_B = k_B - \mathsf{sk}_B \cdot e$

$s = s_A + s_B$ ⟷ $s = s_A + s_B$

A Simple Attempt

sk_A sd_A

sd_B sk_B



has collected

$k_A = F(\mathsf{sd}_A, m)$ | $k_B = F(\mathsf{sd}_B, m)$

$R_A = k_A \cdot G$ | $R_B = k_B \cdot G$

$R = R_A + R_B$ ⟷ $R = R_A + R_B$

$e = H(R \| m)$ | $e = H(R \| m)$

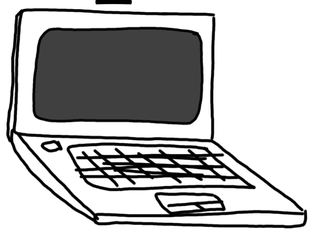
$s_A = k_A - \mathsf{sk}_A \cdot e$ | $s_B = k_B - \mathsf{sk}_B \cdot e$

$s = s_A + s_B$ ⟷ $s = s_A + s_B$

A Simple Attempt

sk_A sd_A

sd_B sk_B



has collected

$s_B = k_B - \mathsf{sk}_B \cdot e$

$k_A = F(\mathsf{sd}_A, m)$ | $k_B = F(\mathsf{sd}_B, m)$

$R_A = k_A \cdot G$ | $R_B = k_B \cdot G$

$R = R_A + R_B$ ⟷ $R = R_A + R_B$

$e = H(R \| m)$ | $e = H(R \| m)$

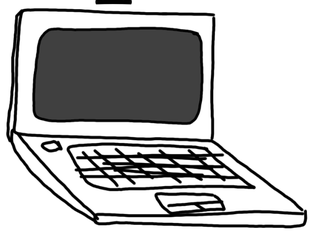
$s_A = k_A - \mathsf{sk}_A \cdot e$ | $s_B = k_B - \mathsf{sk}_B \cdot e$

$s = s_A + s_B$ ⟷ $s = s_A + s_B$

A Simple Attempt

sk_A sd_A

sd_B sk_B



has collected

$s_B = k_B - \mathsf{sk}_B \cdot e$

$k_A^* = F(\mathsf{sd}_A, m)+1$

$R_A = k_A \cdot G$

$R = R_A + R_B$ ⟷ $R = R_A + R_B$

$e = H(R\|m)$ $\quad$ $e = H(R\|m)$

$s_A = k_A - \mathsf{sk}_A \cdot e$

$k_B = F(\mathsf{sd}_B, m)$

$R_B = k_B \cdot G$

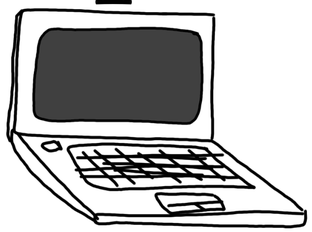
$s_B = k_B - \mathsf{sk}_B \cdot e$

$s = s_A + s_B$ ⟷ $s = s_A + s_B$

A Simple Attempt

sk_A sd_A

sd_B sk_B



has collected

$s_B = k_B - \mathsf{sk}_B \cdot e$

$k_A^* = F(\mathsf{sd}_A, m) + 1$

$R_A^* = k_A^* \cdot G$

$R = R_A + R_B$ ⟷ $R = R_A + R_B$

$e = H(R \| m)$ $\quad$ $e = H(R \| m)$

$s_A = k_A - \mathsf{sk}_A \cdot e$

$k_B = F(\mathsf{sd}_B, m)$

$R_B = k_B \cdot G$

$s_B = k_B - \mathsf{sk}_B \cdot e$

$s = s_A + s_B$ ⟷ $s = s_A + s_B$

A Simple Attempt

sk_A sd_A

sd_B sk_B

has collected

$s_B = k_B - \mathsf{sk}_B \cdot e$

$k_A^* = F(\mathsf{sd}_A, m) + 1$

$R_A^* = k_A^* \cdot G$

$k_B = F(\mathsf{sd}_B, m)$

$R_B = k_B \cdot G$

$R^* = R_A^* + R_B$ ⟷ $R^* = R_A^* + R_B$

$e = H(R \| m)$

$e = H(R \| m)$

$s_A = k_A - \mathsf{sk}_A \cdot e$

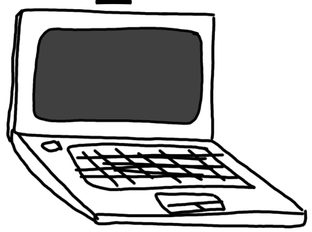
$s_B = k_B - \mathsf{sk}_B \cdot e$

$s = s_A + s_B$ ⟷ $s = s_A + s_B$

A Simple Attempt

$\mathsf{sk}_A \quad \mathsf{sd}_A$

$\mathsf{sd}_B \quad \mathsf{sk}_B$



has collected

$s_B = k_B - \mathsf{sk}_B \cdot e$

$k_A^* = F(\mathsf{sd}_A, m) + 1$

$R_A^* = k_A^* \cdot G$

$k_B = F(\mathsf{sd}_B, m)$

$R_B = k_B \cdot G$

$R^* = R_A^* + R_B$ ⟷ $R^* = R_A^* + R_B$

$e^* = H(R^* \| m)$ $\qquad$ $e^* = H(R^* \| m)$

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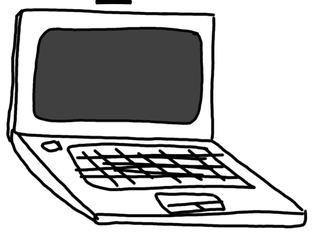

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sk_A sd_A

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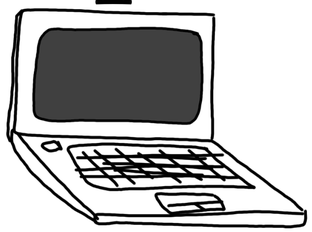

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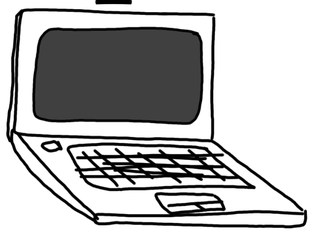

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has collected

$s_B = k_B - \mathsf{sk}_B \cdot e$

$s_B^* = k_B - \mathsf{sk}_B \cdot e^*$

Two linear equations in two unknowns

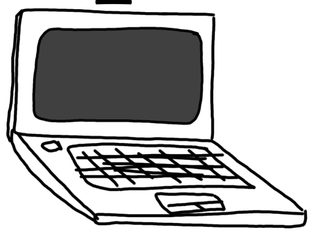
Simply solve for sk_B and reconstruct sk

[Maxwell Poelstra Seurin Wuille 19]

A Simple Attempt

sk_A sd_A

sd_B sk_B



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General flavour of problem previously encountered in Resettable ZK/MPC

[Canetti Goldreich Goldwasser Micali 00]

Maintain a Counter?

- **Simple proposal**: each device maintains a counter (CTR), and derives for eg. $k_A = F(sd_A, CTR)$. Each time CTR is accessed it is incremented, k_A is always fresh

- However this introduces a new attack surface: CTR reuse would be catastrophic

- Undetectable reuse of stale state is a significant concern in practice, due to:
  - Power supply interruptions
  - Restoring from backups
  - Virtual Machines loaded with old snapshots

Can be adversarially induced, or even due to careless mistakes

Isn’t this a Systems problem?

- Maintaining persistent state (‘state continuity’) is difficult even on dedicated hardened and isolated devices [Parno Lorch Douceur Mickens McCune 11]

- Broadly two flavours of general systems solutions:
  1. **Use ‘helper’ nodes**. Inapplicable to our dishonest majority setting.
  2. **Special Purpose Hardware** [PLDMM 11][Strackx Piessens 16]
     - Qualitatively, this induces extra physical assumptions
     - Quantitatively, trusted hardware is slow (60-100ms to increment an Intel SGX Trusted Monotonic Counter), expensive, and has a limited lifespan as non-volatile memory can wear out in a few days of continuous use [Matetic Ahmed Kostiainen Dhar Sommer Gervais Juels Capkun 17]

Can it be a Crypto problem?

- In this work we study whether solving this as a cryptographic protocol design problem can be of benefit:

How can we design a threshold Schnorr protocol that enjoys stateless deterministic signing?

…no device need sample fresh randomness, **or rely on updating state** after (Distributed) KeyGeneration

Can safely restore crashed devices with long-term secrets

- We develop new techniques to construct Schnorr threshold signing that is **stateless and deterministic by design** while relying on **native cryptographic tools** that we estimate to be significantly **faster than trusted hardware**

Intuition

$\mathsf{sk}_A \quad \mathsf{sd}_A$

$\mathsf{sd}_B \quad \mathsf{sk}_B$

$k_A = F(\mathsf{sd}_A, m)$

$R_A = k_A \cdot G$

$R = R_A + R_B$ ⟷ $R = R_A + R_B$

$e = H(R \| m)$

$s_A = k_A - \mathsf{sk}_A \cdot e$

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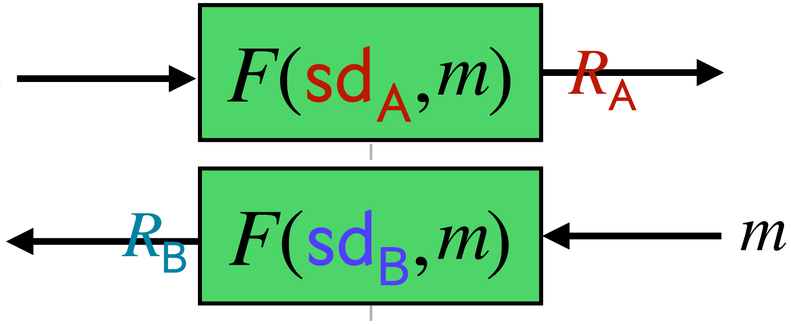

$R = R_A + R_B$

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$s_A = k_A - \mathsf{sk}_A \cdot e$

$s = s_A + s_B$

$R = R_A + R_B$

$e = H(R \| m)$

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$s = s_A + s_B$

Instantiating $F(\mathsf{sd}, m)$

- **Canonical instantiation**:
  - Produce $\mathsf{Com} = \mathsf{Commit}(\mathsf{sd})$ during Distributed KeyGeneration
  - When signing m, prove in ZK that public parameters (Com, R, F, m) satisfy the relation $R = F(\mathsf{sd}, m) \cdot G$ where $\mathsf{Decommit}(\mathsf{Com}, \mathsf{sd}) = 1$

- There is a myriad of ways to construct such a proof system. We prioritise:
  - **Conservative assumptions**: F must be a standard PRF since security of the signature (which is exposed to the outside world) strongly depends on F
  - **Lightweight computation**: friendly to weaker devices (eg. mobile cryptocurrency wallet) as well as institutional high-throughput settings
  - **Round efficiency**: match regular threshold Schnorr (3 rounds)

Our Approach

- Variety of candidate cryptographic tools to instantiate $F(\mathsf{sd}_A, \cdot)$:
  - Succinct Non-interactive Arguments of Knowledge (SNARKs)
  - ~~Generic Multiparty Computation~~
  - MPC-in-the-head
  - Garbled Circuits

- Recall our constraints:
  - Standardised choice for F (eg. AES, SHA)
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  - Match round efficiency of Threshold Schnorr

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[Nick Ruffing Seurin Wuille 20]

Custom PRF+Bulletproofs very bandwidth efficient (~1KB), heavy to execute (~1 second)

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Zero-knowledge for "composite statements"

[Nick Ruffing Seurin Wuille 20]

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ZK for Composite Statements

- **Garbled Circuits** and **MPC-in-the-head**: lightweight proof systems that are traditionally efficient for Boolean Circuits, but not algebraic operations
- Framework of [Chase Ganesh Mohassel 16] for Garbled Circuits, extended to MPC-in-the-head by [Backes Hanzlik Herzberg Kate Pryvalov 19] bridges these respective techniques with algebraic operations with good concrete efficiency
- This is a great direction, since standardised tools (AES, SHA) have compact Boolean Circuit representations, and we require support for Elliptic Curve algebra
- Our target: only cheap symmetric key operations per proof, but existing techniques do not achieve this out of the box

This Work

- In existing works [CGM16, BHHKP19] applied to our setting the dominant cost (computation and bandwidth*) lies in the logistics of the Boolean-algebraic bridge, and in encoding the witness sd
- In particular they require $\Theta(\kappa)$ exponentiations, due to homomorphic commitments and Committed Oblivious Transfer (C-OT) per bit of the witness
- [CGM16] show how to replace the commitments by garbling $\tilde{O}(\kappa^2)$ extra gates, which concretely is more expensive than standard PRFs ($8 \times$ AES)
- We focus on the garbled circuit approach [CGM16] and develop new techniques so that the **dominant cost of a proof is garbling and evaluating the PRF F** which is quite efficient for traditional block ciphers like AES

Our Approach

- We make use of the Zero-Knowledge from Garbled Circuits (ZKGC) paradigm [Jawurek Kerschbaum Orlandi 13] augmented by a conditional-disclosure round compression technique [Ganesh **K** Patra Sarkar 18]

- We develop the following new techniques to tailor and improve this paradigm:
  - Garbling gadget to output the exponentiation of an encoded input
    - Improves $\tilde{O}(\kappa^2)$ Boolean gate gadget [CGM16] to $O(\kappa)$ Boolean gates
    - Also applies to anonymous credentials based on ECDSA [CGM16]
  - Custom C-OT protocol to preprocess all public-key operations
    - Input encoding now cheaper than garbling F
    - Also applies to Distributed Symmetric Encryption [Agrawal Mohassel Mukherjee Rindal 18]

Garbling Gadget

$$\varphi(y_1 y_2 \ldots y_\eta) = y \cdot G$$

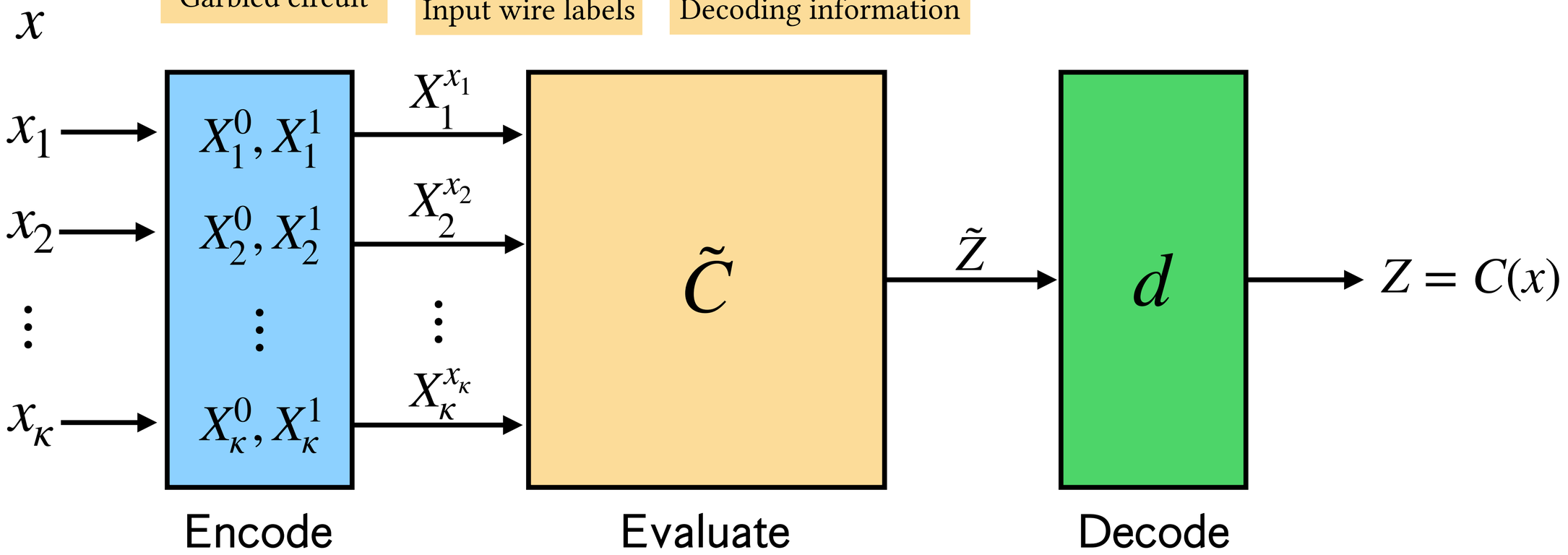
- **Task**: garble $C(x) = \varphi(F(x))$ where $F : \{0,1\}^\kappa \to \{0,1\}^\eta$, and $\varphi : \{0,1\}^\eta \to \mathbb{G}$

  Boolean circuit — Exponentiation / Curve multiplication

- **Our approach**:

$$\tilde{C}, \{X_i^0, X_i^1\}_{i\in[\kappa]}, d \leftarrow \mathsf{Garble}(C)$$

Garbled circuit — Input wire labels — Decoding information



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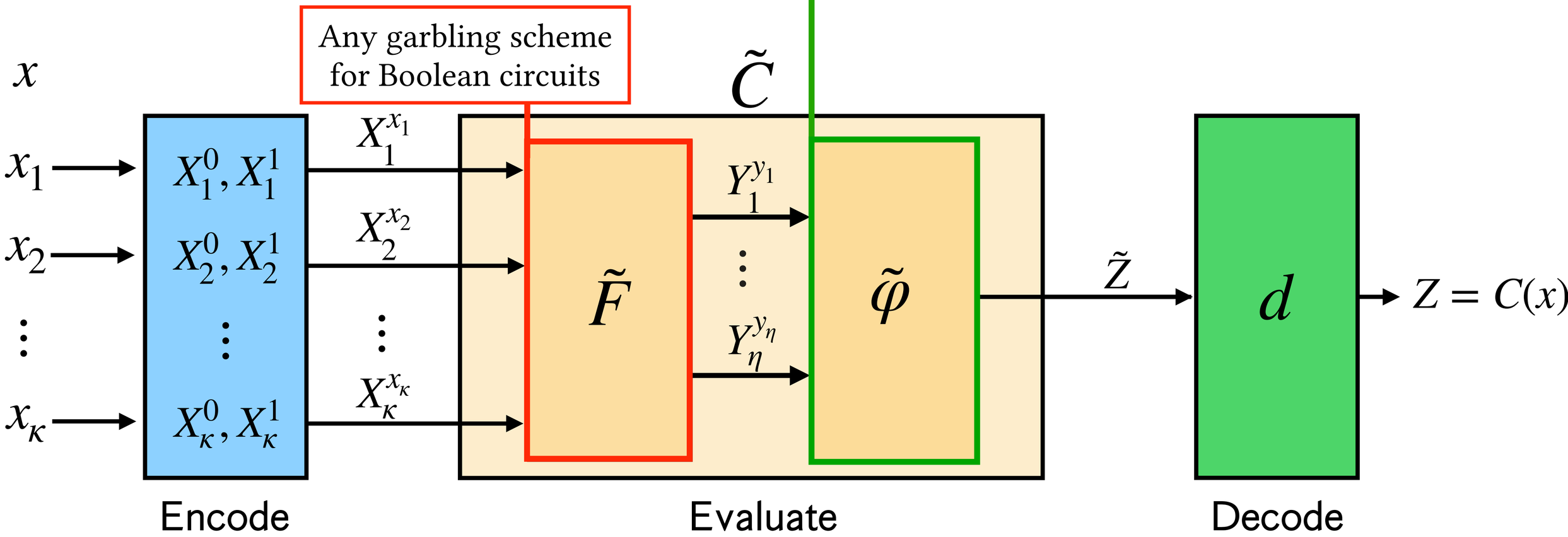
  - F: Boolean circuit
  - φ: Exponentiation / Curve multiplication
- **Our approach**: New gadget inspired by Oblivious Linear Evaluation of [Gilboa 99]



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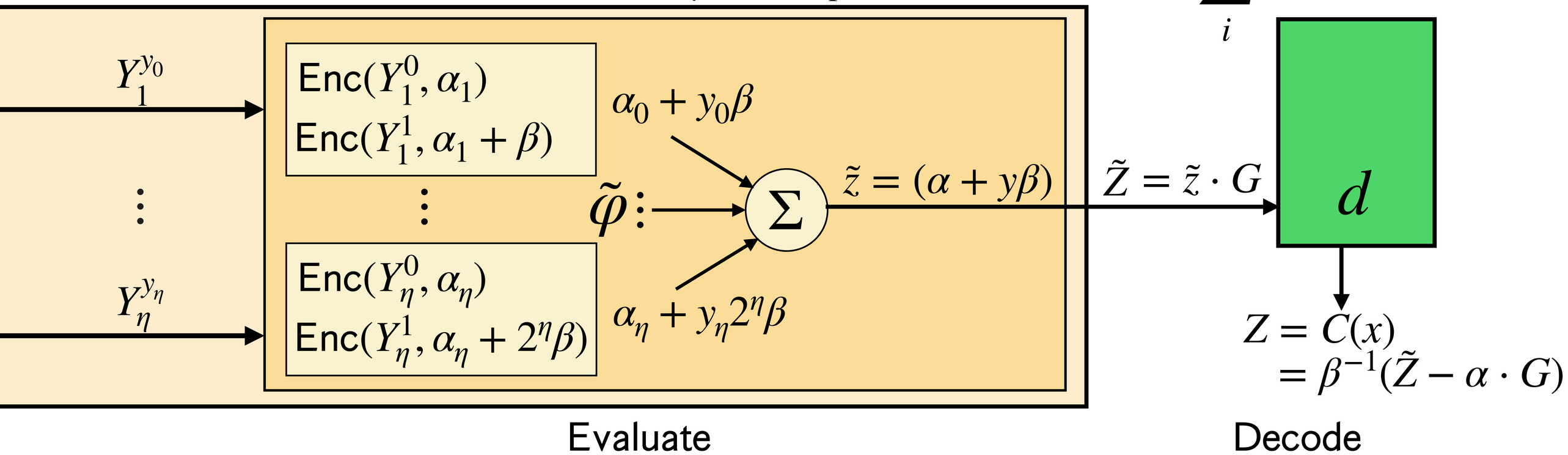
  - F: Boolean circuit
  - φ: Exponentiation / Curve multiplication
- **Our approach**: New gadget inspired by Oblivious Linear Evaluation of [Gilboa 99]

$$(\beta, \alpha_1, \alpha_2, \cdots, \alpha_\eta) \leftarrow \mathbb{Z}_q^{\eta+1}, \text{ and set } \alpha = \sum_i \alpha_i$$



Garbling Gadget

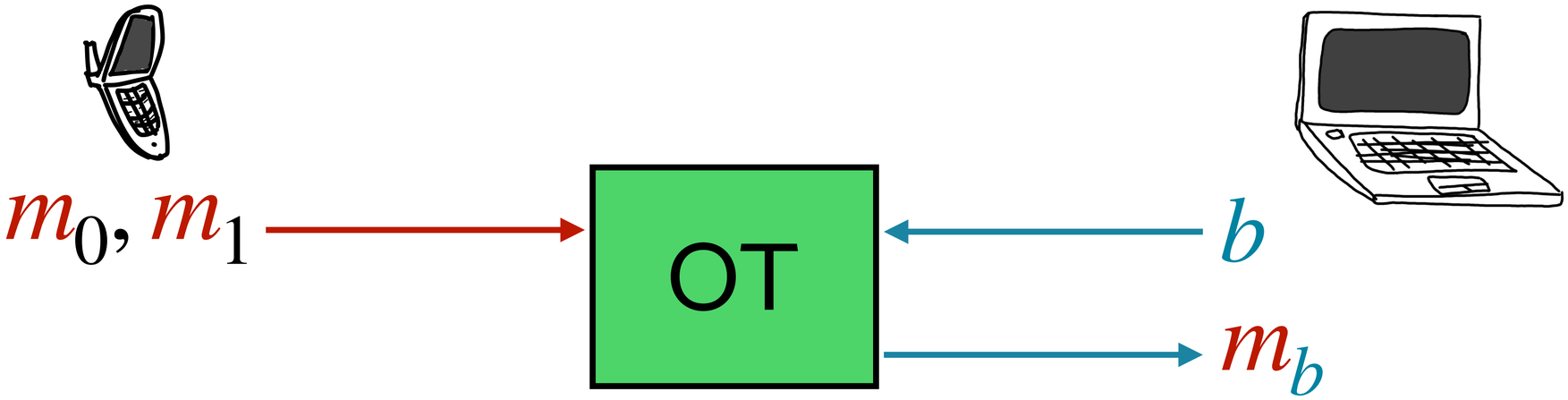
- Security intuition:
  - **Soundness/Authenticity**: (α, β) act as information-theoretic MAC key — $\tilde{Z}^* = \alpha + \beta y^*$ is hard to forge
  - **Zero-knowledge/Uniqueness**: Once $(\alpha, \beta), Z$ are fixed, one can simulate $\tilde{Z}$ perfectly; $\tilde{Z} = \beta^{-1} \cdot (Z - \alpha \cdot G)$ resembles simulation of a Schnorr signature
- **Efficiency**: η cipher texts of size $\log_2(q)$ bits each — equivalent to garbling $2\log_2(q) \in O(\kappa)$ Boolean AND gates (compare with garbling $\tilde{O}(\kappa^2)$ [CGM16])

Concrete improvement: $60\times$ in computation and bandwidth

Cost is now insignificant compared to garbling PRF circuit

Committed OT

- Executed to encode each bit of input to the garbled circuit [JKO13]



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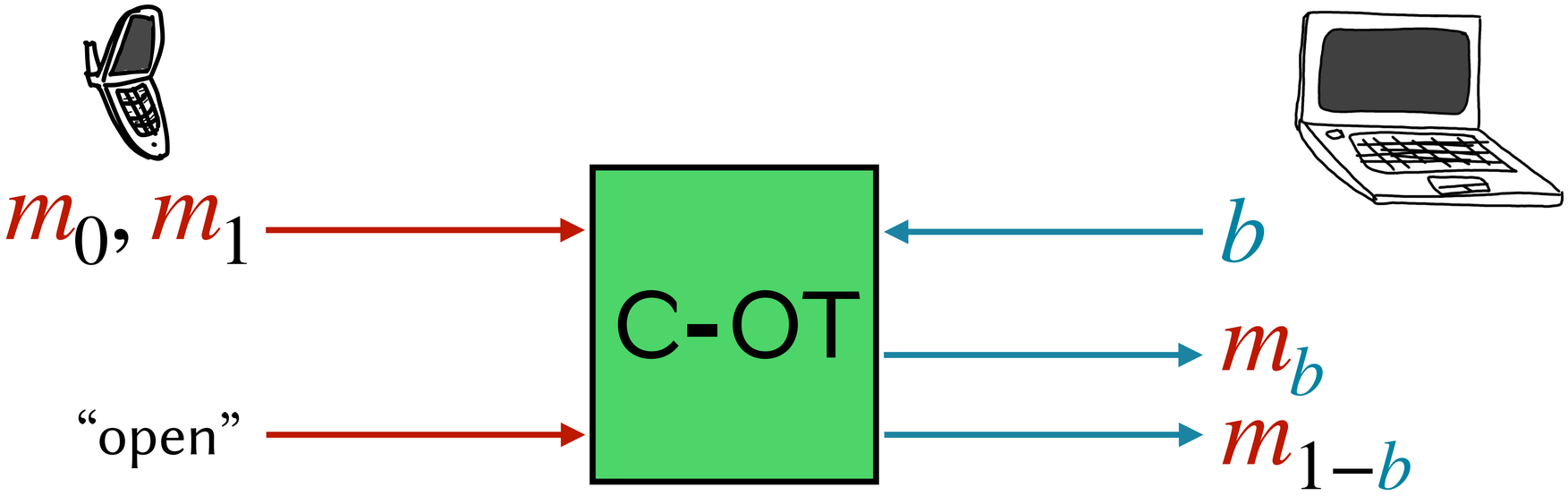

- Unclear how to preprocess public key operations for C-OT; standard OT Extension does not natively support decommitting messages without sacrificing setup
- **Relaxation**: input b once and transmit an unbounded number of message pairs

C-OT: Naive Attempt

$\mathsf{sd}_0, \mathsf{sd}_1$ → OT ← b

OT → sd_b

x

$p_0 = \mathsf{PRF}(\mathsf{sd}_0, x)$
$p_1 = \mathsf{PRF}(\mathsf{sd}_1, x)$

$\mathsf{ct}_0 = p_0 \oplus m_0$
$\mathsf{ct}_1 = p_1 \oplus m_1$ →

Retrieve
$m_b = \mathsf{ct}_b \oplus \mathsf{PRF}(\mathsf{sd}_b, x)$

"open"
p_0, p_1 →

$m_{1-b} = \mathsf{ct}_{1-b} \oplus p_{1-b}$

C-OT: Naive Attempt

$\mathsf{sd}_0, \mathsf{sd}_1$

OT

0

sd_0

$b = 0$

x

$p_0 = \mathsf{PRF}(\mathsf{sd}_0, x)$
$p_1 = \mathsf{PRF}(\mathsf{sd}_1, x)$

$\mathsf{ct}_0 = p_0 \oplus m_0$
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p_0, p_1

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C-OT: Naive Attempt

$\mathsf{sd}_0, \mathsf{sd}_1$

OT

0

sd_0

x

$p_0 = \mathsf{PRF}(\mathsf{sd}_0, x)$

$p_1 = \mathsf{PRF}(\mathsf{sd}_1, x)$

$\mathsf{ct}_0 = p_0 \oplus m_0$

$\mathsf{ct}_1 = p_1 \oplus m_1$

Retrieve

$m_0 = \mathsf{ct}_0 \oplus \mathsf{PRF}(\mathsf{sd}_0, x)$

"open"

p_0, p_1^*

$m_1 = \mathsf{ct}_1 \oplus p_1$

C-OT: Naive Attempt

$\mathsf{sd}_0, \mathsf{sd}_1$ → OT ← 0

OT → sd_0

x

$p_0 = \mathsf{PRF}(\mathsf{sd}_0, x)$
$p_1 = \mathsf{PRF}(\mathsf{sd}_1, x)$

$\mathsf{ct}_0 = p_0 \oplus m_0$
$\mathsf{ct}_1 = p_1 \oplus m_1$ →

Retrieve
$m_0 = \mathsf{ct}_0 \oplus \mathsf{PRF}(\mathsf{sd}_0, x)$

"open"
p_0, p_1^* →

$m_1^* = \mathsf{ct}_1 \oplus p_1^*$

Tool: UC Commitments

- We make use of Universally Composable commitments [Canetti 01] to construct our C-OT scheme

- Specifically, UC commitments that permit the following algorithms:
  - $\mathsf{Setup}(1^\kappa) : (\mathsf{td}, \mathsf{ck}, \mathsf{vk})$
    - Trapdoor
    - Verification key
    - Commitment key
  - $\mathsf{Commit}(\mathsf{ck}, m) : \mathsf{Com}, d$
    - Decommitment information
  - $\mathsf{Verify}(\mathsf{vk}, \mathsf{Com}, m, d) : \{0,1\}$
  - $\mathsf{Extract}(\mathsf{td}, \mathsf{Com}) : m$
    - **Straight-line algorithm** that extracts the committed message
    - Conventionally a “proof artefact” that we actually execute in our construction

C-OT from UC Commitments

Setup

$\mathsf{ck}_0, \mathsf{ck}_1$

b

$\mathsf{td}_b, \mathsf{vk}_0, \mathsf{vk}_1$

$\mathsf{ct}_0, d_0 \leftarrow \mathsf{Commit}(\mathsf{ck}_0, m_0)$
$\mathsf{ct}_1, d_1 \leftarrow \mathsf{Commit}(\mathsf{ck}_1, m_1)$

$\mathsf{ct}_0, \mathsf{ct}_1$

Retrieve
$m_b = \mathsf{Extract}(\mathsf{td}_b, \mathsf{ct}_b)$

"open"

$(m_0, d_0), (m_1, d_1)$

$\mathsf{Verify}(\mathsf{vk}_0, \mathsf{ct}_0, m_0, d_0)$
$\wedge\ \mathsf{Verify}(\mathsf{vk}_1, \mathsf{ct}_1, m_1, d_1)$

C-OT from UC Commitments

- Security **follows directly from straight-line extraction** and equivocability of the UC commitment scheme

- Instantiation of the commitment scheme borrows ideas from literature on UC Commitments from Error Correcting Codes
  [Cascudo Damgård David Giacomelli Nielsen Trifiletti 15]

- Committing, Verification, and Extraction require only PRF evaluations (and some hashing) which translates to the online complexity of C-OT

- **Concretely** for 128-bit computational and 60-bit statistical security, all C-OT instances combined cost ~16k PRF invocations (+some hashing) and ~82KB in bandwidth, which is roughly the **same cost as garbling and evaluating a single AES circuit**

In Summary

- The ZKGC paradigm [JKO13] is well suited to enabling stateless determinism in Threshold Schnorr when prioritising computational efficiency and standard assumptions
- The dominant cost of previous techniques [CGM16] applied to this setting were in input encoding (C-OT) and garbling exponentiations
- Our new techniques for these tasks improve their efficiencies to the point where the **dominant cost is now garbling and evaluating the PRF circuit**
- Our cost analysis estimates computation to be faster than using trusted hardware
- See the paper for: concrete cost analysis, optimisations and tricks, details

Thanks!

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Thanks **Eysa Lee** for